

ARISTOTELIAN EQUATIONS OF HOLOGRAPHY

the hologram output includes terms with the following phases:

$$\begin{aligned} \Phi_{\text{out}}(x,y) &= \Phi_{\text{obj}}(x,y) - \Phi_{\text{ref}}(x,y) + \Phi_{\text{ill}}(x,y) && \text{, and (direct image)} \\ &\quad -\Phi_{\text{obj}}(x,y) + \Phi_{\text{ref}}(x,y) + \Phi_{\text{ill}}(x,y) && \text{, and (conjugate image)} \\ &\quad \Phi_{\text{ill}}(x,y) && \text{(zero-order image)} \end{aligned}$$

or, in general form:

$$\Phi_{\text{out}}(x,y) = m (\Phi_{\text{obj}}(x,y) - \Phi_{\text{ref}}(x,y)) + \Phi_{\text{ill}}(x,y), \quad m = 0, \pm 1, \pm 2, \dots$$

it follows that, for the diffraction angle:

$$\sin \theta_{\text{out}} = m \frac{\lambda 2}{\lambda 1} (\sin \theta_{\text{obj}} - \sin \theta_{\text{ref}}) + \sin \theta_{\text{ill}}$$

and, for the y-focus distance (horizontal focus):

$$\frac{1}{R_{\text{out,h}}} = m \frac{\lambda 2}{\lambda 1} \left(\frac{1}{R_{\text{obj}}} - \frac{1}{R_{\text{ref}}} \right) + \frac{1}{R_{\text{ill}}}$$

and for the x-focus distance (off-axis, when astigmatism arises):

$$\frac{\cos^2 \theta_{\text{out}}}{R_{\text{out,v}}} = m \frac{\lambda 2}{\lambda 1} \left(\frac{\cos^2 \theta_{\text{obj}}}{R_{\text{obj}}} + \frac{\cos^2 \theta_{\text{ref}}}{R_{\text{ref}}} \right) + \frac{\cos^2 \theta_{\text{ill}}}{R_{\text{ill}}}$$