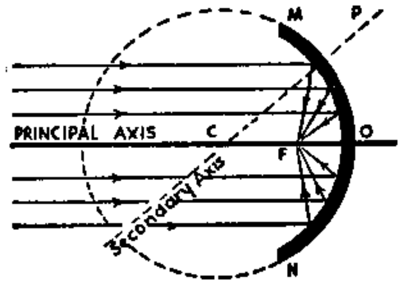
OPTICAL ENGINEERING NOTE #75
APPLICATIONS OF THE CONJUGATE RATIOS OF CONCAVE MIRRORS

Solving the Simple Mirror Formula* for a variety of object distances yields the corresponding image distances. The combination of object and image distances are called **conjugate pairs**, as the two distances work together.

The **conjugate ratio**, image distance divided by object distance, is also known as the **magnification**. There are several conjugate ratio zones, and they and their applications are categorized below.



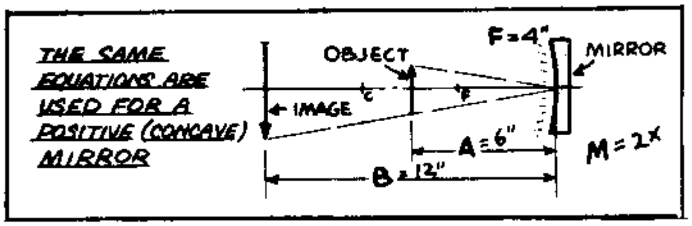
OBJECT AT INFINITY CONJUGATE RATIO, Case 0**: This graphic also illustrates the concept of focal length. Rays from a point on an object, so far away that there is essentially no angle between the top and bottom rays entering the lens, will be reunited after travelling all that way at a distance one focal length away from the lens.

For spherical mirrors, the focal length is one half the radius of curvature of the sphere that the mirror is but a small section of. Aspheric shapes like parabolas, ellipses, hyperbola all have foci defined mathematically.

Plugging in infinity as the object distance in the magnification equation*** means we end up dividing by it, and the mathematical consequences means we have zero magnification****. Yet

---

*Same as the **Simple Lens Formula**,
1/A + 1/B = 1/F,
A = object distance,
B = image distance,
F = focal length.

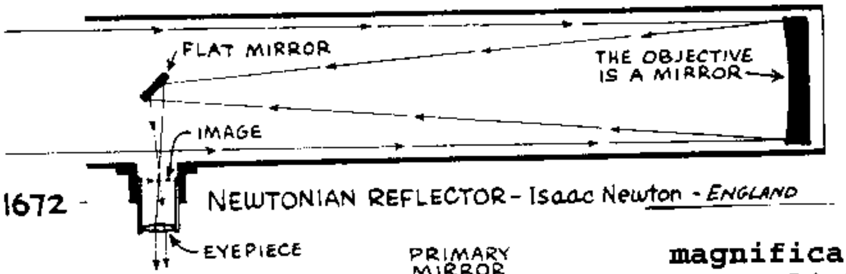


The source book for **Figures 1 -5, Vitalized Physics, Robert H. Carleton, College Entrance Book Company, 1960 were already numbered, so the prelude was assigned 0.

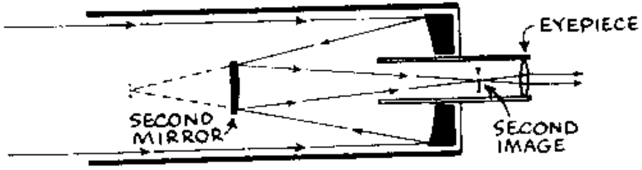
***magnification = image distance divided by object distance.

**** "Any number divided by infinity = 0."

projecting an image of the sun with a telescope mirror or any concave mirror shows that there is some dimension, with longer focal length mirrors providing larger images. To find the magnification at this particular conjugate ratio the angular magnification must be computed, as discussed in the **Handout, MAGNIFICATION**.



1672 - NEWTONIAN REFLECTOR - Isaac Newton - *ENGLAND*



1672 - CASSEGRAIN REFLECTOR - Guillaume Cassegrain - *FRANCE*

The most obvious optical device that uses this <u>infinite conjugate ratio</u> is the telescope objective. (Figure A.) Angular magnificaton of these rays is accomplished by the telescope's eyepiece, which is placed its focal length away from the image delivered by the objective, an example of an **Image at Infinity, Case 4** application.

Telescope objectives are usually not spherical but paraboloid, to eliminate spherical aberration. The Hubble Space Telescope's primary mirror was not ground to the proper curved shape, introducing spherical aberration into the system which gave fuzzy edges to objects.

Since the top and bottoms of laser beams are parallel to each other to a first order approximation, this conjugate ratio can be applied to the spreading of them. This is the method favored by Dr. Tung Jeong in his latest pamphlet[1] on holography. (Figure B.) The advantage of using a front-surfaced spherical mirror method of spreading beams is that there is only one surface to gather dust which create horrible bulls-eye artifacts in the enlarged beam. However, cleaning a front-surface mirror is much riskier compared to a lens as a simple scratch in the coating renders the optic useless.

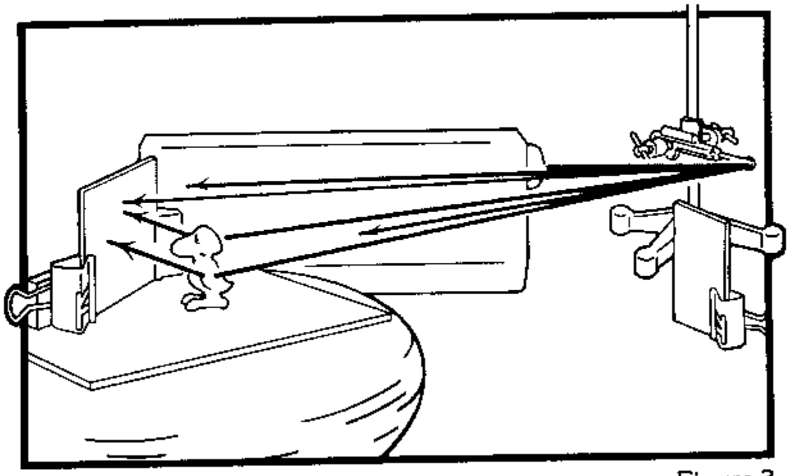
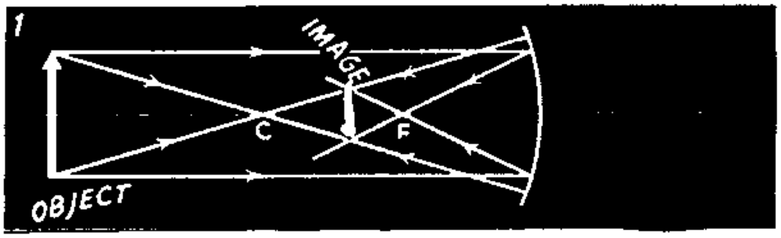


Figure 3

For small Helium-Neon lasers there is no problem with the beam coming to a focus one focal length away from the aluminized mirror surface. Applying this technique to a higher powered laser would result in destruction of the simple aluminum coating or even breakdown of the air due to the intense concentration of energy at the focus. Hard dielectric thin film interference coatings could handle the stress of the incident beam, but the only way to prevent breakdown of the air is to not let it focus really in space but to let it diverge from a virtual source, by using a convex mirror.

In any case, spreading a laser beam with a mirror inevitably produces something other than a truly cone-shaped divergence, because the angle of incidence is not along the optical axis. As shown in Figure B, the angle of incidence is greater than zero for the divergent beam to clear the laser. The greater the angle, the greater the degree of astigmatism. There will be two foci separated in space for the horizontal and vertical planes. For quick and clever holographic demonstrations this is no real problem, but hopelessly inadequate for reference beams for anything serious.

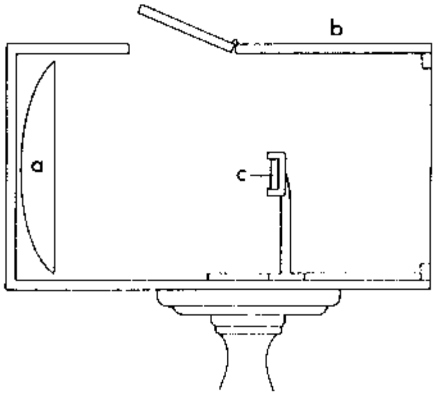
FINITE CONJUGATE RATIOS, Case 1: This is the typical picture taking zone for lenses, but it is rare to find mirrors in photographic objectives. The object is greater than 2f away and the image is formed between 1f and 2f behind the lens. Object distance > image distance, so the magnification < 1, meaning the object is minified.



The first patented photographic camera uses a concave mirror as the imaging optic. Invented by Alexander Wolcott[2], these cameras (Figure C.) focussed the image of the subject onto a 2 by 2 1/2" Daguerreotype plate.* The mirror, fabricated by the American telescope maker, Henry Fitz[3], was of 8 inches diameter and 12 inches focal length (for an f/# of 1.5, incredibly fast for its day!).

---

*See the **Handout, A GUIDE TO IDENTIFYING NINETEENTH CENTURY PHOTOGRAPHS IN THE FIELD.**

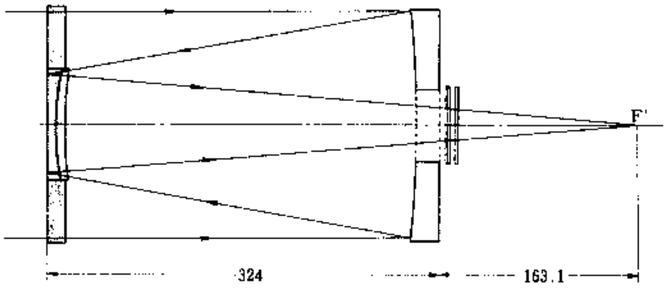


The plateholder was placed in the focus of the concave mirror, and it is in the path of the incoming object rays. However, since rays from each object point arrive at the complete surface of the imaging optic, the central rays will not be missed. But the plateholder, acting as a stop, does decrease the light gathering power of the the mirror, dropping the intensity at the plane of the light-sensitive plate.

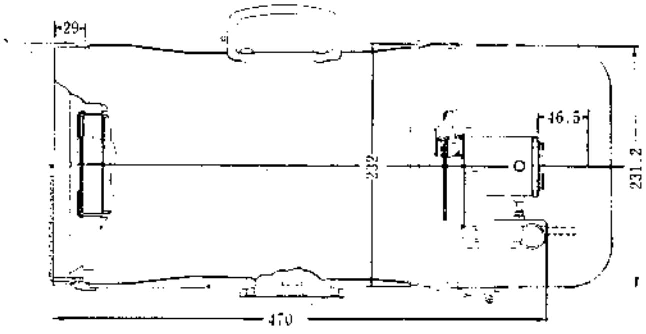
A more contemporary use of a concave mirror in photographic cameras is as the collecting optic for <u>reflex-telephoto objectives</u>. (Figure D.) By folding the image path back and forth through the optical housing, extremely long focal lengths for telephotographic lenses can be housed in extremely short packages.

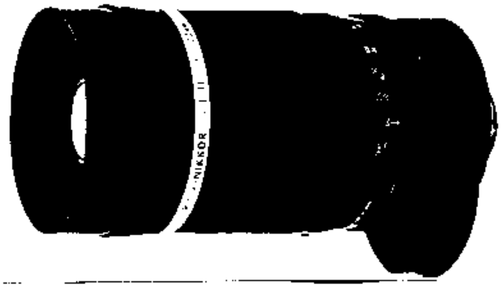
REFLEX-NIKKOR 1:11 f = 1000mm

Type: 4 groups of 6 elements
Aperture: 11
Diaphragm: Fixed diaphragm; exposure controlled with filters and shutter speeds.
Focusing range: 25 ft. (8m)~Inf.
Angle of view: Diagonal: 2° 30′
Horizontal: 2°
Vertical: 1° 20′
Focusing screens: Rec.: A,B,C,D,E,F,J
Also use: H3
Attachment/filter size: 108mm, (P=0.75)
Filters: 34.5mm L39, Y48, O56 and R60, built-in on turret mount.
NOTE: Other filters not to be used, since filters furnished with lens are designed with a thickness that will not affect focus.
Weight: 1.6k (56 oz.)
Camera body can be rotated 90°.



Optical formula of Reflex-Nikkor 1:6.3 f=1000mm.

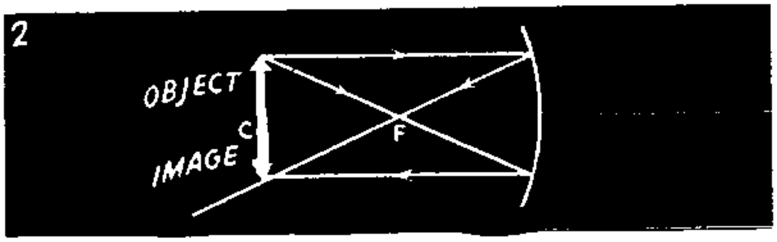




Dimensions of Reflex-Nikkor 1:6.3 f=1000mm.

Every time you observe your reflection in the concave side of a spoon you are witnessing an application of this conjugation.

UNIT MAGNIFICATION, Case 2: Object and Image positions are symmetric about the lens; the most common application is in Xerox machines, to make a copy that is the same size as the orignal. The object and image are both two focal lengths away from the principal plane of the lens, or in the plane of the center of curvature



There is a major drawback to using a mirror in this conjugate position. Notice that the object and image in **Figure 2** are next to each other in the same plane, otherwise the image is formed right on top of the object. The imaging system must have the input entering from the side, and then the dreaded off-axis aberrations of astigmatism, coma, and distortion enter into the image. Nevertheless, there are many models of xerographic copiers that use mirrors or combinations of lenses and mirrors in this configuration.

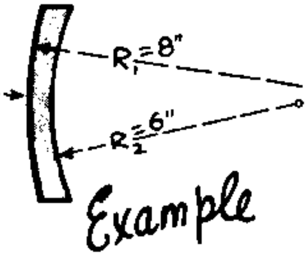

"FOLDED" LENS

Used, Plummer convex glass lens assembly with mirrored back. Specifications unknown at this time. Image [illegible] lens then is reflected back out thru lens. Flat field corrected. Used for copy machine & laser printers. [illegible] 4" mounting flange. Overall size [illegible] 2" dia. X 2" thick. Mounted in removable aluminum bracket.

6303-LN $6.95 WT 1.9

Because of the issue of aberrations, a mirror like the one illustrated in the catalog clipping^4 is actually a lens with a mirror glued onto the back side of it. (Figure E.) It could be considered a radical example of a <u>Mangin Mirror</u>, (Figure F), a non-front surfaced mirror which has different radii of curvature for the inside and outside surfaces, resulting in focussing by refraction at the beginning, focussing by reflection at the second surface, and finally focussing again by refraction at the retro-reflected output.

MANGIN MIRROR
... IS A NEGATIVE MENISCUS LENS SILVERED ON CONVEX SIDE. A POSITIVE (CONVERGING) MIRROR WILL RESULT IF F.L. OF LENS IS GREATER THAN THE RADIUS OF ITS CONVEX SURFACE

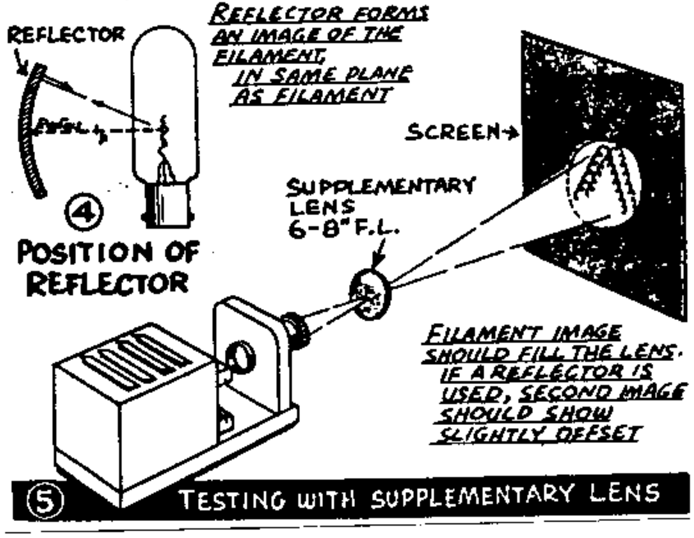


$$\text{F.L. LENS} = \frac{R_1 \times R_2 \times 2}{R_1 - R_2} = \frac{8 \times 6 \times 2}{8 - 6} = \frac{96}{2} = 48''_{\text{NEGATIVE}}$$

$$\text{F.L. MIRROR} = \frac{R_1}{2} = \frac{8}{2} = 4''_{\text{POSITIVE}}$$

$$\text{F.L. COMB.} = \frac{\frac{1}{2}F_{LENS} \times F_{MIR}}{\frac{1}{2}F_{LENS} - F_{MIR}} = \frac{24 \times 4}{24 - 4} = \frac{96}{20} = 4.8''_{\text{POSITIVE}}$$

There is a common on-axis application of this conjugate ratio in the form of reflectors for light bulbs used in projection systems. Light radiates from a lamp filament in all directions, and most of it is headed in a direction opposite that of the intended destination, the transparency to be illuminated. (Figure G.)



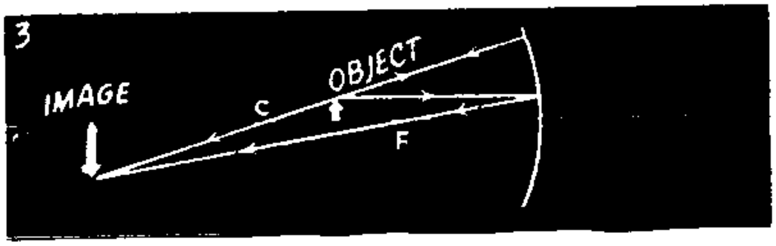
Placing the filament of a lightbulb at the center of curvature of a mirror means that the rays travel to the mirror's surface along a radius, which is at right angle to the mirror, so that the light comes right back onto itself.

Standard practice for implementing reflectors behind bulbs includes a bit of "slop", for if the filament were imaged back onto itself, no light would go in the desired direction. The bulb and mirror are slightly askew to each other, so that the image if the filament is formed inbetween the coils of the filament, and then pass through the glass envelope fo the bulb onward to the condensing lens system.

Finding the focal length of a spherical mirror can be easily done with a divergent laser beam by positioning the beamspreader in such a position with respect to the mirror so that the image of the spatially coherent laser spot coincides with the actual spot. (This is best accomplished with a pinhole placed at the real focus of a positive diverging lens.) When this is done, measure the distance from the pinhole to the mirror, and you have the radius of curvature of the mirror. Divide by 2, and the quotient is the focal length.

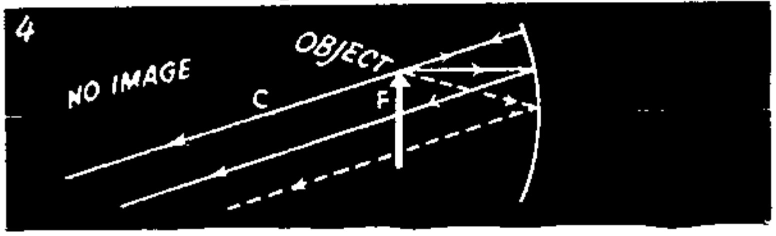
The definition of f/# is focal length divided by the size of the aperture. But this is true only for the infinite conjugate ratio. The light-gathering power of a lens or mirror is greatly diminished at this 1:1 ratio or other ratios near it, as the true f/# is really the focal distance divided by the diameter of the aperture. At unit magnification the focal distance is twice that of the infinite one, so the f/# is doubled, meaning that the intensity at the film plane is one-fourth of the light expected at the marked f/#. (For example, a marked f/2 becomes f/4 at this magnification.)

MAGNIFICATION, Case 3: The object is placed between 1f and 2f, and the image is thrown far away from the mirror and is larger than life. Notice the symmetry between this case and **Case 1**; the object and image distances are interchanged.



The Sega "hologram" arcade game[5] game uses this technique to form a real image of a video monitor, so that the image floats in space, separate from any physical apparatus.

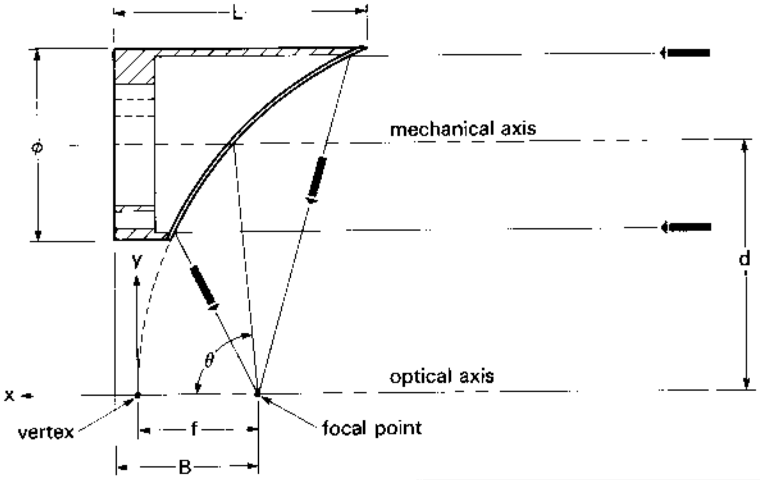
IMAGE AT INFINITY CONJUGATE DISTANCE, Figure 4: Since the infinitely far parallel rays focus 1f away from the lens, the image rays of an object placed 1f from a lens will exit parallel. The image will be formed at infinity.



Collimators, which make parallel beams of light, work at this conjugate ratio, either using white light issuing from a pinhole or a spatially filtered laser beam. It is, in essence, a telescope in reverse.

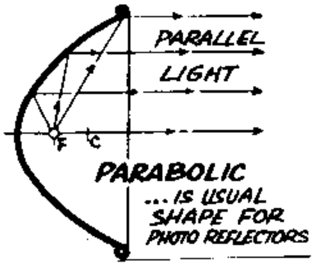
Placing the laser and beamspreader along the optical axis results in a shadow in the center of the beam from the equipment. Even using a small diagonal mirror like in a <u>Newtonian telescope</u>[6] will produce some sort of doughnut beam profile. The usual solution is to have the point source incident at an angle larger than the normal, so that entering and exiting rays clear each other, like in the beamspreading application of **Case 1's** Figure B.

If a mirror were to be used most effectively as a high performance laser beam collimator, an off-axis parabolic design works best, (Figure H), as the astigmatism of a spherical mirror used off-axis as suggested above will allow one plane of the exiting bundle of rays to be parallel, while the other plane will not be



parallel but will focus somewhere, either in a real or virtual sense. (In front of or in back of the mirror.)

These off-axis parabolic reflectors, since they are cut from larger pieces of mirror with the focus on-axis, are quite expensive. But if the application warrants <u>diffraction-limited</u> performance, then there is no other choice.



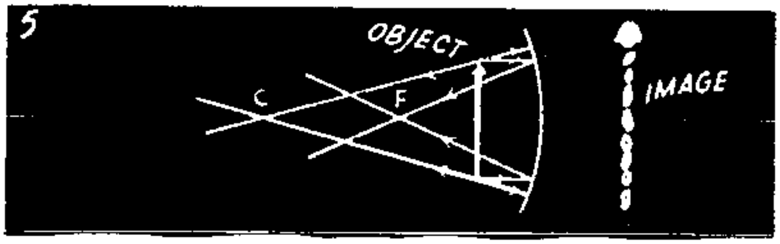
The <u>dichroic</u> reflectors found behind the typical quartz-halogen MR-16 sized bulb are somewhat parabolic in shape, with the bulb filament at the focus of the parabola. (Figure I.) But since the filament has dimension, there are many differently-angled parallel bundles exiting the assembly from each point of the glowing coil. The lamp's far field intensity distribution pattern exhibits some spread thanks to this factor, plus the reflectors themselves are not smooth but faceted, introducing more deviant angles.

Additionally there is light leaving the bulb in the forward direction, without being directed by the reflector, and it radiates in all directions as from a point source, further widening the illuminated area. Oftentimes there will appear a black hole in the center of the spread beam, making it look like a doughnut of light, due to the shadow of the nipple on the end of the light bulb.

Mag-lites and other focussing flashlights pass the bulb through positions in front of the reflector which encompass **Infinite, Magnified**, and **Positive** conjugate relationships, which will image the filament onto the target of interest at a wide range of distance, or spread the light out for a flood effect.

An interesting imaging technique using the infinite conjugate distance is utilized in the wonder toy of science, the Mirage. For more details, see the **Handout, Optical Enginerring Note #, THE SECRETS OF THE MIRAGE REVEALED**.

POSITIVE CONJUGATE DISTANCE, Figure 5: If an object is positioned less than 1f away from the mirror, a <u>virtual</u> image is formed that is further behind the mirror than object is in front of it. Unlike the convex mirror whose virtual image is always smaller than the



object, this is a <u>magnified</u> virtual image. The most obvious use of this case is in cosmetic or shaving mirrors, where blemishes, etc. can be seen in more detail.

When describing this case for lenses, it was called **NEGATIVE CONJUGATE DISTANCE**. But in this case, it is called **POSITIVE** as the image is formed virtually to the right, or positive number side of the optic.

A beautiful demonstration of all of the conjugate cases of the **Simple Mirror Formula** can be performed with the diverging cone of light generated by passing a laser beam through a lens or spatial filter as mentioned above, and moving the point source nearer and further from the mirror and watching the image spot react. When the diverging source is less than 1f away, (**Case 5**), no real image of the spot is formed, but looking into the mirror reveals a spot further behind the mirror than the diverger is in front. It will also be noted that the angle of divergence is smaller in the reflected beam than in the incident one.

Concave mirrors have other applications; how many have you encountered?