# Wavefront Reconstruction with Continuous-Tone Objects*

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(Received 24 April 1963)

Holograms and high-quality reconstructions have been made by using a two-beam interferometric technique. The extraneous twin image and other interfering terms have been eliminated. Two types of objects have been used which are not suitable for the conventional wavefront reconstruction technique: objects which do not transmit a strong background wave (e.g., transparent lettering against a dark background) and continuous-tone objects.

## 1. INTRODUCTION

A TWO-STEP imaging process developed by Gabor[1] consists of photographing the Fresnel diffraction pattern of an object and using this recorded pattern to construct an image of the original object. The recorded diffraction pattern, called a hologram, bears little resemblance to the original object, but contains most of the information needed to reconstruct the object. When the hologram is illuminated with monochromatic light derived from a point source, the interaction of the hologram transparency with the incident light generates diffracted wavelets, a component of which is identical to the wavelets which were used to produce the hologram. This component can be used to reconstruct an image of the original object.

A primary difficulty with the process is that the phase of the incident beam is lost in the recording process. This, in general, makes a reconstruction impossible, except for the special class of object in which the major portion of the incident beam is transmitted without scattering. The scattered light combines vectorially with the direct beam to produce a resultant whose phase differs only slightly from that of the direct beam alone. Here the loss of phase is of relatively less importance and does not prohibit a recognizable reconstruction. Even in this case, however, the loss of phase exacts its toll, in that not more than half of the light transmitted by the hologram contributes to the reconstructed image. A second component, with energy equal to that of the desired component, forms an extraneous image whose Fresnel diffraction pattern is superimposed on the desired image, causing its degradation. Another component, which represents intermodulation distortion, contributes further to the degradation.

The conventional Gabor technique is thus limited to relatively simple objects which transmit a large proportion of the light without scattering. For example, transparencies which contain dark lettering against a transparent background are well suited to the process and to the authors' knowledge, are the only kind which have appeared in the published literature. The reverse of this, an object with a dark background, is wholly unsuited to the process. Continuous-tone objects also are unsuitable because the extraneous terms generated in the process are sufficiently large as to completely obscure the reconstructed image.

In the 15 years since Gabor's announcement of the process, various published papers have described means for eliminating the extraneous twin image. In particular, a technique described by the present authors[2] removes not only the extraneous image term, but also the intermodulation distortion term and, in addition, tends to reduce other distortion terms such as may arise from nonlinearities in the film transmittance–exposure characteristic. The process, in theory, permits perfect reconstructions for any type of object transparency. Initial experimental results, as shown in the authors' previous paper, demonstrated the correctness of the ideas presented, but the reconstructions obtained were lacking in sharpness and were generally of low quality, owing to a lack of equipment. This condition has been corrected and the potentialities of the process have now been realized.

## 2. MAKING THE HOLOGRAM

The technique was described briefly in the previous paper[2] and is reviewed here with emphasis on a descriptive approach. A two-beam interferometric process is used in producing the hologram, as shown in Fig. 1. The object, located at plane $P_1$, is illuminated with collimated monochromatic light, and a Fresnel diffraction pattern of the object is formed at plane $P_2$. Adjacent to the object is a prism. The portion of the incident beam

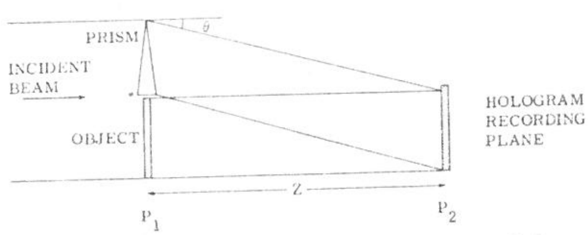



FIG. 1. Wedge technique for producing a two-beam hologram. The object is placed in the lower part of plane $P_1$; the hologram is recorded at plane $P_2$.

* This research was sponsored by the A. F. Office of Scientific Research of the Office of Aerospace Research, as part of the Advance Research Projects Agency's Vela Uniform Program. It was presented at the March 1963, Jacksonville meeting of the Optical Society of America [J. Opt. Soc. Am. 53, 522 (1963)].

[1] D. Gabor, Nature **161**, 777 (1948); Proc. Roy. Soc. (London) A197, 454 (1949).

[2] E. Leith and J. Upatnieks, J. Opt. Soc. Am. **52**, 1123 (1962).

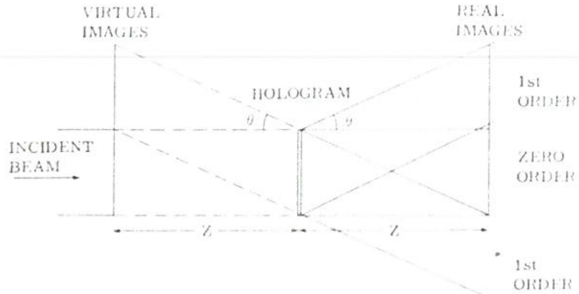



FIG. 2. The reconstruction process. Low-quality conventional reconstructions occur on hologram axis. High-quality reconstructions occur in the first-order diffracted waves.

which impinges on the prism is deviated through an angle $\theta$ and thereby becomes superimposed on the lower portion of the beam. The superposition of the two beams results in a fringe pattern which is superimposed on the Fresnel diffraction pattern of the object. A photographic plate at $P_2$ records the resultant pattern, thereby producing the hologram.

In the absence of the upper beam, the photographic plate at $P_2$ produces a conventional Gabor hologram. Let the amplitude of the light at $P_2$ be given by

$$U(x,y)=A(x,y)e^{i\phi(x,y)}, \qquad (1)$$

where $A$ is the amplitude modulus and $\phi$ is the phase of the impinging light. The photographic plate records only the magnitude factor $A$; the phase portion $e^{i\phi}$ is discarded. The conventional hologram is thus an incomplete record.

The interference pattern produced when the second beam, which we call the reference beam, is present results in a hologram in which the phase portion $\phi$ of the Fresnel diffraction is also recorded. Let the reference beam have an amplitude modulus $A_0$. This beam produces at $P_2$ a wave of amplitude $A_0e^{i\xi_c x}$, where the phase term $e^{i\xi_c x}$ results from the beam impinging on $P_2$ at an angle. A beam impinging on a plane at an angle $\theta$ produces (for small values of $\theta$) a progressive phase retardation $\exp i2\pi\theta x/\lambda$ across this plane. Hence we have the relation $\xi_c=2\pi\theta/\lambda$.

When the reference beam is present, the light amplitude distribution at the hologram recording plane is $A_0e^{i\xi_c x}+Ae^{i\phi}$. Let us assume that the plate which records this distribution has a response which is linear with intensity, that is, suppose the amplitude transmittance of the plate after development to be given by

$$T=T_0-kI, \qquad (2)$$

where $I$ is the intensity distribution at plane $P_2$,

$$I=|A_0e^{i\xi_c x}+Ae^{i\phi}|^2, \qquad (3)$$

d $k$ are constants determined by the trans- ...posure characteristic of the plate. Equation ...eral, a reasonable approximation to the ...ristic over a transmittance between ... measured relative to the base transmittance. The resultant tra... ...ttance of the recording plate is, therefore,

$$T=T_0-k|A_0e^{i\xi_c x}+Ae^{i\phi}|^2$$
$$=T_0-kA_0^2-kA^2-2kA_0A\cos(\xi_c x-\phi). \qquad (4)$$

The plate thus behaves like a square-law modulating device, producing a term $2kA_0A\cos(\xi_c x-\phi)$ which is the real part of the original Fresnel diffraction pattern, modulated onto a carrier $\xi_c$. In the absence of the diffracting object this term represents a uniform fringe pattern produced by the interference between the two beams. When the diffracting object is present, its Fresnel diffraction pattern modulates this fringe pattern. The amplitude modulus of the diffraction pattern produces an amplitude modulation of the fringes, and the phase portion $\phi$ produces a phase modulation (or spacing modulation) of the fringes.

It appears, then, that this process has permitted the photographic plate to record both the amplitude modu-

(a)

(b)

(c)

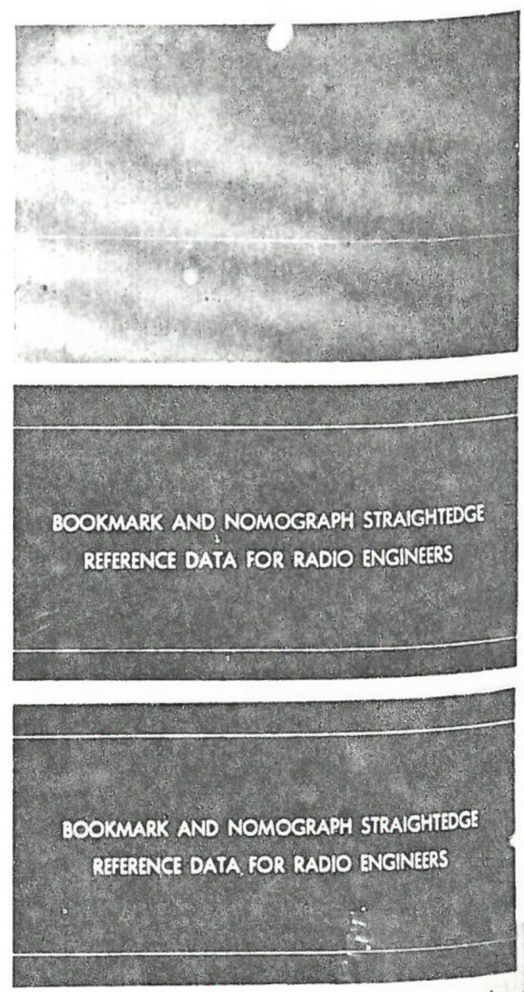



FIG. 3. Hologram and reconstruction of transparent lettering on dark background. (a) Hologram; (b) reconstruction; (c) original object. The original object was about 1.5 cm in length; the distance between objects and hologram planes was about 2 ft. The hologram was made with Kodak Spectroscopic Plates, Type 649-F.

(a)

(b)

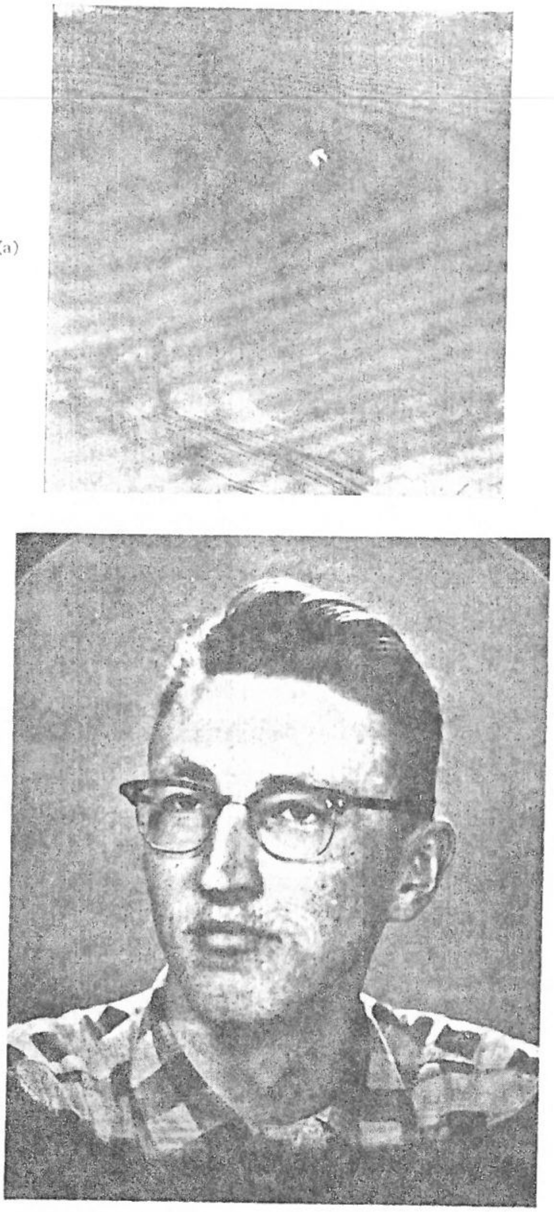


FIG. 5. Hologram and reconstruction of a portrait. The object was about 1.5 cm square and the distance between object and hologram was about 2.5 ft. The hologram was made with Kodak Spectroscopic Plate, Type 649-F.

image a distance $z$ from the hologram. The factor $e^{i\xi_c x}$ alters this view only in that it results in the virtual image being displaced laterally a distance proportional to $\xi_c$. The conjugate term $\frac{1}{2}kA_0Ae^{-i(\xi_c x-\phi)}$ produces the real image, which likewise is displaced from the axis, as im- 'ied by the factor $e^{-i\xi_c x}$.

## DISCUSSION OF THE TECHNIQUE

...ding results are based on the square-law ... of the recording plate, as given by Eq. ...ion is only approximately obtained, there ...der distortion terms present on the hologram. These will, for the most part, give rise to second and higher-order diffracted waves, which in the reconstruction process will form additional images at greater off-axis positions, and will therefore be separated from the first-order images. Hence, while we have assumed a specific and only approximately realized film characteristic, the actual characteristic is not at all critical to the process, and in no case was it necessary, or apparently even desirable, to consider controlling this characteristic.

By controlling the relative amplitude of the object-bearing beam and the reference beam, for example, by the use of attenuating filters placed in one of the beams, the contrast of the fringe pattern can be controlled. If this contrast were made sufficiently small by attenuating the object-bearing beam, then Eq. (2) could certainly be made to hold to great accuracy, if this were desired. However, if the fringe contrast is too low, the reconstructed image will tend to be grainy. Good reconstructions are in practice possible over a wide range of fringe contrasts.

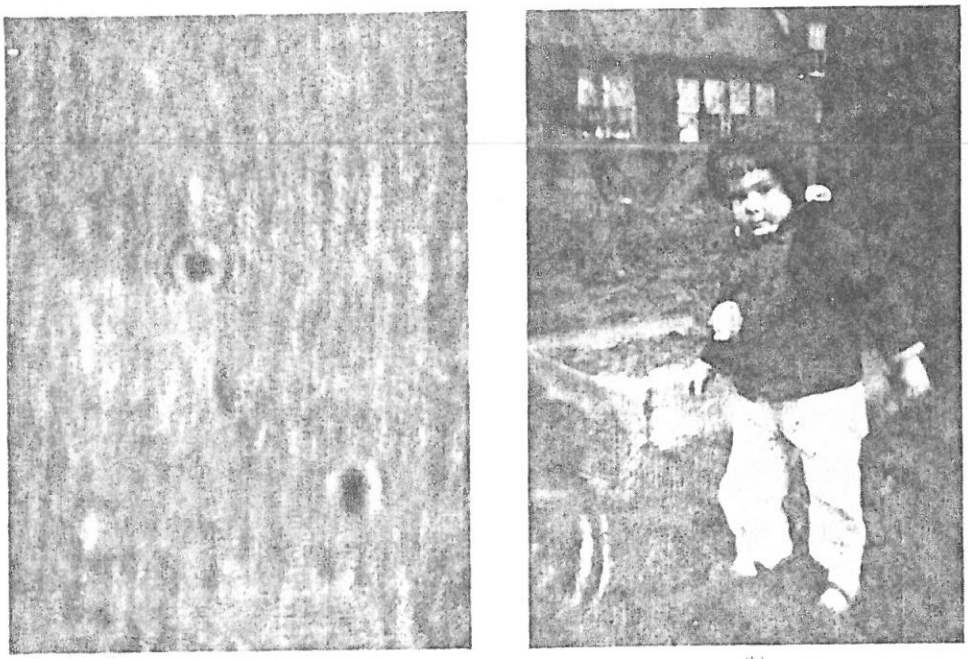
The reconstructed image that results has the same contrast as the original object, irrespective of the gamma of the recording plate, or of the manner in which the plate is developed. The reason for this is that all spatial frequency components of the object, including the dc or bias term, are preserved on the hologram in their proper proportion, except for whatever minor effects are produced by the modulation transfer function of the hologram plate. Such is not the case for the conventional hologram, in which the reconstruction includes an ambient background or dc term which tends to reduce the contrast of the reconstructed image, and whose magnitude is a function of the recording process.

Another feature of interest is that the reconstructed image is a positive, that is, it has the same polarity as the original object. If the hologram is contact printed so as to produce a negative of the original hologram, then this negative hologram also produces a positive reconstruction. This situation is different from the conventional wavefront reconstruction process, in which the hologram produces a negative reconstruction, and the contact-printed copy of the hologram produces a positive image.

## 5. EXPERIMENTAL RESULTS

The effectiveness of the process is shown by Figs. 3–5. Figure 3 shows a hologram and reconst. ıction of transparent lettering against a dark background. Such an object is not suitable for the conventional type of hologram, since no strong background wave is produced. However, the object is quite s... ...le for the two-beam process. The reconstruction ... very nearly equal in quality to the original, which is shown for comparison.

Figures 4 and 5 show holograms and reconstructions of continuous-tone transparencies. This type of object is fully an order of magnitude more difficult to reconstruct than simple objects like lettering. Figure 4 shows a



(a) (b)

FIG. 4. Hologram and reconstruction of a scene. The object was about 1.5 cm in height, and the distance between object and hologram was about 4 ft. The hologram was made with Kodak High Contrast Copy.

tus and the phase of the Fresnel diffraction pattern. However, the complete demonstration of this requires that the final term of Eq. (4) be separable from the remaining terms. Also, for our purpose there is the additional problem of how this term can be used to reconstruct the original object.

## 3. RECONSTRUCTION

The various terms of Eq. (4) can indeed be separated and a reconstruction achieved. There are two different, although somewhat similar, methods for doing this. The more general one, described in Ref. 2, requires forming the Fraunhofer diffraction pattern, or spatial frequency spectrum, of the hologram by the use of a lens, and then carrying out a spatial filtering procedure. The other method is more simple in that it does not require such spatial filtering, but requires a larger value of the spatial carrier frequency, $\xi_c$.

The second method of reconstruction is used here. The reconstruction is made in much the same way as with conventional holograms. The hologram is placed in a collimated beam of monochromatic light, as shown in Fig. 2. The bias term $T_0-kA_0^2$ and the term $kA^2$ combine to form a reconstruction that is essentially the reconstruction produced by the conventional hologram, in which a real image forms at a distance $z$ on one side of the hologram, and a virtual image forms at an equal distance on the other side of the hologram. The distance $z$, of course, is just the distance that existed between the object and the recording plate when the hologram was produced, assuming light of the same wavelength is used throughout. Both the real and virtual images form about the optical system axis and for this reason the two images are inseparable, i.e., each image must be viewed with the other as a background. In this region there occur also the extraneous intermodulation terms described previously[2] and prevent reconstructions of good fidelity from all but simple objects.

The fine-line structure of the hologram, embodied in the term $kA_0A\cos(\xi_c x-\phi)$, causes the hologram to act like a diffraction grating, producing a pair of first-order diffracted waves, as shown in Fig. 2. One of these produces a real image, occurring in the same plane as the conventional real image, but displaced to an off-axis position. Similarly, the other diffracted wave produces a virtual image, situated in the plane of the conventional virtual image, but displaced. As seen from Fig. 2, the light components comprising these two off-axis images are nonoverlapping, and both components are removed from the region where the conventional reconstructions occur. These images are of high quality, and are free from the effect of any of the extraneous terms which the reconstruction produces. Either the real or virtual image can be photographed, but it is more convenient to use the real image, since this can be recorded by placing a plate at the image position, thus avoiding the need for a lens. Hence, the entire process is carried out without lenses.

A comprehensive analysis supporting the above contentions about the reconstructions was given previously[2] and is not repeated here. However, we show the plausibility of the contentions. If the term $kA_0A\cos(\xi_c x-\phi)$ of Eq. (4) is rewritten in its exponential form, $\frac{1}{2}kA_0Ae^{i(\xi_c x-\phi)}+\frac{1}{2}kA_0Ae^{-i(\xi_c x-\phi)}$, it is seen that the first exponential term is, to within a constant multiplier and an exponential term $e^{i\xi_c x}$, exactly the complex function that describes the Fresnel diffraction pattern produced at plane $P_2$ by the object. This term can therefore be considered as having been produced by a virtual scene containing a number of objects which are quite recognizable in the reconstruction, although the hologram is unintelligible.

Figure 5 shows the hologram and the reconstruction of a portrait; this is the most difficult type of object for wavefront reconstruction. There is considerable resolution present in the picture (which, of course, is not entirely preserved in the publication process). However, the difficult part of the reconstruction is not in the fine detail, but in the broad, uniform areas, which constitute a considerable part of Fig. 5. These will appear mottled in the reconstruction unless considerable care is taken; the hologram must be extremely clean, free from defects (scratches, etc.) and if made on film instead of plate, must be placed in a liquid gate containing a fluid which closely matches the index of refraction of the film. The need for the liquid gate arises from the thickness variations of the hologram film. The effect of film-thickness variations in this type of work is discussed by Ingalls.[3]

The holograms shown in Figs. 3–5 warrant some discussion. First, it should be pointed out that, in addition to what is reproduced in the figures, there is a fine-line fringe structure which is completely lost in the publication process. Most of the visible structure is noiselike and is unrelated to the useful data contained on the hologram. This feature is characteristic of two-beam holograms. The hologram of Figs. 3 and 5 show prominent coarse fringes. These result from interference between the incident beam and reflected light from the back surface of the recording plate. Their presence has no relevance to the recorded data, but neither do they noticeably degrade the reconstructed image.

The holograms and reconstructions were made using a helium–neon gas laser operating at 6328 Å. The light produced by the laser is highly monochromatic and can be imaged to a fine point; these features make the laser an excellent light for this application.

Holograms and high-quality reconstructions have also been made using a conventional mercury arc lamp in combination with a Mach–Zehnder interferometer (as also suggested in Ref. 2). This technique appears to work as well as the method used here, but is more difficult to carry out experimentally.

[3] A. Ingalls, Phot. Sci. Eng. 4, 135 (1960).

## 6. COMMENTS AND FUTURE POSSIBILITIES

The results shown here should encourage development of practical applications of the wavefront reconstruction process. In addition, there are many other directions in which investigation can proceed. One possibility is the production of the generalized hologram described previously.[2] Presently the authors are working on a color process of wavefront reconstruction. Another interesting facet is the superposition of several holograms on a single plate by means of multiple exposures. The various reconstructed images will be separated if the fringe pattern of each hologram has a different orientation, since the reconstructed images will then be displaced in different directions. This process has been demonstrated with three overlapping holograms, without noticeable deterioration of the reconstructed images. Another possibility is to make the hologram and the subsequent reconstruction in highly divergent light beams. This will produce a reconstruction which is a magnified image of the original object. This process constitutes a lensless microscope which should be highly corrected and work over a large field. Moderate magnifications (about 10) have been demonstrated, and magnifications far greater than this seem attainable.

## ACKNOWLEDGMENT

The authors wish to acknowledge the assistance of B. A. Vander Lugt, who, in performing work of a related nature, showed the authors the value of a gas laser for a light source. While the laser is not essential to the process, its considerable brilliance, spatial coherence, and monochromaticity immensely facilitated the work.