XIV.—Experiments in Diffraction Microscopy.* By G. L. Rogers,†

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(With Two Plates and Nine Text-figures)

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Synopsis

Experiments have been performed, using purely optical methods, to verify and extend the theory of Gabor's diffraction microscope. An elementary theory of the process is first given, from which certain generalizations are provisionally drawn. In particular, a focal length is attributed to any Fresnel diffraction pattern and the hologram derived from it by photography. The variation of this focal length with wavelength and scale factor is postulated by analogy with a zone-plate, and the power-rate for a hologram is defined. These deductions are then verified by experiment, and a summary is given at the end of § 10. Various other confirmatory experiments are then described.

Adequate information is given about apparatus and technique to enable new entrants into this field to obtain satisfactory results with the minimum of preliminary trial.

§ 1. Introductory

In a number of papers (Gabor, 1948, 1949) Gabor has described a system of microscopy in coherent light whereby a magnified but indistinct image of an object may be obtained without a lens, and a "reconstructed" or more distinct image later produced with an auxiliary lens. In particular, it is proposed to perform the first stage electronically, with a lensless electron microscope, and the second stage optically with a luminous source and suitable reconstructing lens.

The present paper describes an experimental study, using optical methods throughout, of this very ingenious idea. Though the application to electron microscopy has not been entirely forgotten, the work has been directed to the method in its own right. As a result a number of useful generalizations have been discovered empirically and verified theoretically which, though doubtless implicit in the many equations of Gabor (1949), can also with profit be stated explicitly in simpler physical terms.

§ 2. Elementary Theory : Fresnel Diffraction

(*a*) Circular Symmetry

Consider a point source of monochromatic light at O and a small scattering object at R. In practice, a dust particle or fine droplet serves

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admirably. Let us consider the Fresnel diffraction pattern at a plane PM due to the direct radiation from O and scattered radiation from R (fig. 1). We take OR perpendicular to PM, with M on OR. Also let $OR=a$, $RM=b$, in accordance with the notation generally used in Fresnel diffraction.

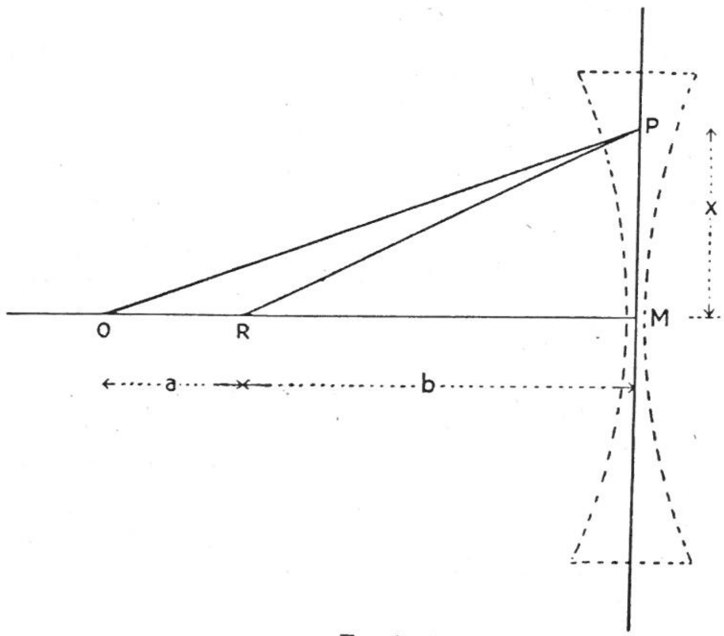


FIG. 1

The direct ray travels along a path OP. If we take $PM=x$, we get:

$$OP^2=(a+b)^2+x^2=OM^2\left(1+\frac{x^2}{(a+b)^2}\right).$$

Suppose now $x<<OM$, then:

$$OP=OM\left(1+\frac{x^2}{2\,.\,OM^2}+\text{higher terms in }\frac{x^2}{OM^2}\right)$$

$$\simeq OM+\frac{x^2}{2\,.\,OM}=a+b+\frac{x^2}{2(a+b)}.$$

Similarly

$$RP\simeq RM+\frac{x^2}{2\,.\,RM}=b+\frac{x^2}{2\,.\,b}.$$

If now we allow for the fact that the scattered light has to go from O to R before it is scattered, and if we assume zero phase-change on scattering, we get the path from O to P via R, *viz.*,

$$OR+RP=a+b+\frac{x^2}{2b}.$$

The path difference between the two disturbances will then be:

$$\Delta = a + b + \frac{x^2}{2b} - a - b - \frac{x^2}{2(a+b)}$$

$$= \frac{x^2}{2}\left[\frac{1}{b} - \frac{1}{a+b}\right].$$

It is now convenient to replace the bracket by the parameter $1/f$ according to the equation:

$$\frac{1}{b} - \frac{1}{a+b} = \frac{1}{f}.$$

It will be seen at once that this parameter has a physical meaning, which experiment shows to be fundamental to the whole process. In short, f *is numerically equal to the focal length of that divergent lens which, placed in the screen position, would image the source in the plane of the object.*

We get now:

$$\Delta = \frac{x^2}{2} \cdot \frac{1}{f}.$$

Since the intensity of illumination at P is a maximum when Δ is an even number of half wavelengths, and minimum if Δ is an odd number of half wavelengths, we get that:

$$\frac{x^2}{2} \cdot \frac{1}{f} = p \cdot \frac{\lambda}{2} \qquad p \text{ even} \rightarrow \text{max.}$$

$$\frac{x^2}{2} \cdot \frac{1}{f} = p \cdot \frac{\lambda}{2} \qquad p \text{ odd} \rightarrow \text{min.}$$

This gives $x^2 = fp\lambda$ as defining a series of rings about M, with maxima at even values of p and minima at odd values of p. The regions between will have intermediate intensities varying in an approximately sinusoidal manner.

If the scattering particle, R, introduces a phase shift, the result is an opening out or closing in of the system of rings, corresponding to the addition of a constant term to Δ. If, for example, there is a phase delay of $\frac{1}{2}\pi$, corresponding to a path increase of $\frac{1}{4}\lambda$, we get:

$$\Delta = \frac{x^2}{2} \cdot \frac{1}{f} + \frac{\lambda}{4} = p\frac{\lambda}{2}$$

and

$$\frac{x^2}{2} \cdot \frac{1}{f} = p \cdot \frac{\lambda}{2} - \frac{\lambda}{4} = q \cdot \frac{\lambda}{2}$$

where

$$q = p - \tfrac{1}{2}$$

The first minimum occurs at $p = 1$ or $q = \frac{1}{2}$ and the first maximum occurs at $p = 2$ or $q = 1\frac{1}{2}$. Thus the rings defined by $q = 1, 2, 3$, etc., which in the previous case were minima or maxima, now become rings of average intensity. Suppose now we replace the sinusoidal variations of intensity by abrupt changes of intensity at the positions $q = 1, 2, 3$, etc., as the intensity moves through the average, so that $I = 0$ from $q = 0 \rightarrow 1$, $2 \rightarrow 3$, $4 \rightarrow 5$, etc., and $I = I_{\max}$ when q goes from $1 \rightarrow 2$ or $3 \rightarrow 4$ or $5 \rightarrow 6$.

Such an arrangement, translated from intensities into densities, is the well-known zone plate.

A zone plate is thus a black and white approximation to the Fresnel diffraction pattern of a small object scattering the incident light with a $\frac{1}{2}\pi$ change of phase.

In particular, if x_1 is the radius of the first ring (where $q = 1$) we have:

$$x_1{}^2 = f\lambda, \quad \text{or} \quad f = \frac{x_1{}^2}{\lambda},$$

the well-known expression for the focal length of a zone plate.

Thus it will be seen that f is the focal length of the zone plate, as well as the parameter of the diffraction pattern. We note further that if O is a source of monochromatic light of wavelength λ, and R be removed, the placing of a zone plate of focal length f in the position of the screen will result in the production of an image at the point R of the source O. The zone plate may thus be said to "reconstruct" the object R in the correct position.

We shall see in general that if the Fresnel diffraction pattern of an object is photographed or otherwise reproduced, and then placed in the original diverging beam, it "reconstructs" the object from the point source in its original position. (This constitutes the Gabor method of diffraction microscopy.) But further, if the reproduction of the Fresnel diffraction pattern (called by Gabor the "hologram") is placed elsewhere in a diverging beam, it will produce a reconstruction of the object in a position related to the source position by the lens formula.

Two other properties of the zone plate deserve attention. First, as is well known, the zone plate can act as both a positive and a negative lens. That is to say, its focal length is strictly speaking $\pm f$. Thus not only does it produce an image of O at R, but also an image of O where a convergent lens of focal length f would form an image of O. There are thus *two* reconstructions of the point R. This has been found by Gabor.

Gabor has shown that the process of photographing the diffraction pattern to form the hologram results in a loss of information; information as to the *phase* of the disturbance in the plane PM. (See also

Bragg, 1950.) The assumption in his method is that if the direct beam is of much greater intensity than the scattered beam, the phases of the resultant disturbance will nowhere differ greatly from that of the direct beam. As the latter is the phase supplied in the reconstruction, this loss of information is, to the first approximation, unimportant. He goes on to show that, as a second approximation, this loss of information results in the production of a secondary image on the side of O remote from R: making the approximation that $OR << RM$, he gets the secondary image at R' where $R'O = OR$. This also holds for a zone plate of focal length $>> OM$. It is clear that the double sign of the zone plate focal length is equivalent to the phenomenon of the hologram in producing two images.

The second property of the zone plate worthy of study is its possession of a whole series of secondary focal lengths, corresponding to higher

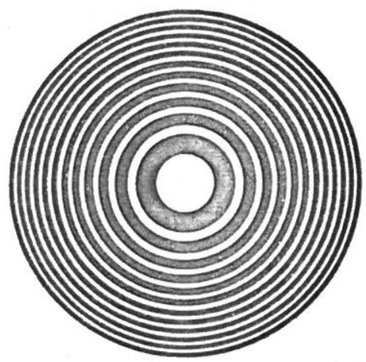


FIG. 2.—Circular zone plate with third order missing from its higher order foci

orders in a diffraction grating. The *powers* of these lenses go up as the odd integers, 1, 3, 5, etc., and may be regarded as higher orders of the fundamental power. It will be seen that the even orders are missing, as in the case of a grating with equal black and white stripes. The reason is in both cases the same. In the first order position, the transparent areas in both devices transmit a range of phases from 0 to π, 2π to 3π, etc., which reinforce. But in all even orders, the range of phases transmitted runs from 0 to $2n\pi$, $4n\pi$ to $6n\pi$, etc., and darkness results. If a zone plate were constructed to transmit only the first third-period zone, blocking the next two-thirds and so forth, it would be found to possess all powers bar multiples of 3. We have verified that all multiples of 3 are absent from the zone plate in fig. 2.

The reason for this lies in the fact that the x values correspond to integral values of p in $\sqrt{(pf\lambda)}$ and hence, say, a trebling of p throughout gives a similar set of x values corresponding to a new focal length

$f' = f/3$. This property may be peculiar to systems of circular or linear symmetry where the x's recur at regular values of $\sqrt{p}$.

§ 3. Elementary Theory : Fresnel Diffraction, (b) Straight Line Symmetry

In the case of an object with a straight line symmetry, a so-called "cylindrical" wavefront is often taken, though to the degree of approximation normally used in these discussions similar results occur with spherical wave-fronts impinging on the straight line object. Thus Jenkins and White (1937), using cylindrical wavefronts and Airy (1877), using spherical wavefronts, both arrive at Fresnel Integrals or the equivalent Cornu spiral as the correct method of solving these problems.

The Fresnel Integrals are tabulated in terms of a dimensionless parameter, v (say), and the minima and maxima of a particular problem may be obtained from the tables in terms of v. In the case of Cornu's Spiral, however, owing to the fact that we plot dimensionless numbers on some particular linear scale, the parameter v acquires the appearance of linear dimensions, and is said to be the length along the arc of the spiral. But this arises from the fact that the scale of the axis is itself a dimensional quantity, to which v is strictly proportional; a fact often overlooked in the discussion of graphical problems.

If O (fig. 1) is the source and R the linear object producing a Fresnel diffraction pattern in the plane PM, we get the positions of minima and maxima in terms of the v values in the tables. These have to be translated into x values in the PM plane. If $OR = a$, $RM = b$, it can be shown that for a wavelength λ, the relation between x and v is (Jenkins and White):

$$x = v\sqrt{\left(\frac{b\lambda(a+b)}{2a}\right)}.$$

If, now, we attempt to express this relation in terms of our parameter, f, we see at once:

$$\frac{1}{f} = \frac{1}{b} - \frac{1}{a+b} = \frac{a+b-b}{b(a+b)} = \frac{a}{b(a+b)}$$

$$f = \frac{b(a+b)}{a}$$

and in particular:

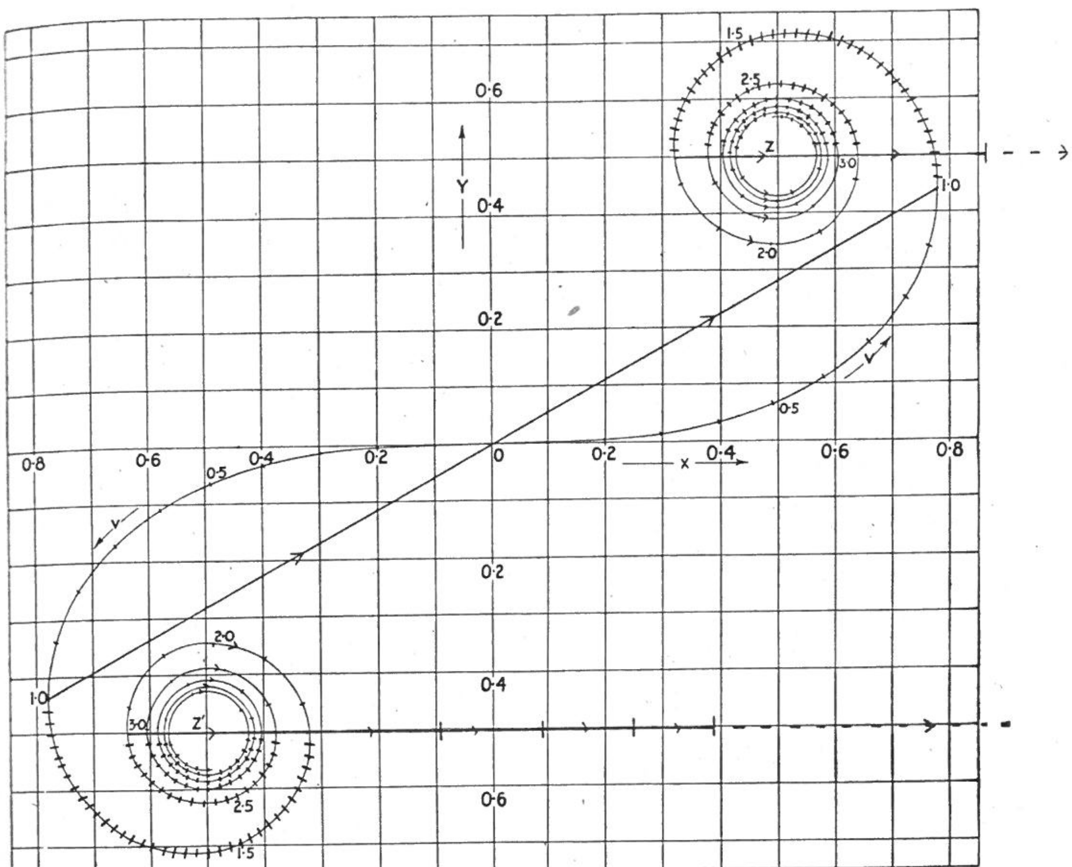
$$x = v\sqrt{\left(\frac{\lambda}{2}\cdot\frac{b(a+b)}{a}\right)} = v\sqrt{\left(\frac{\lambda f}{2}\right)}$$

We see, therefore, that the *scale* of the pattern is, as before, a linear function of $\sqrt{(\lambda f)}$ where f is the focal length of a lens, placed at the screen, to image the source on the object. The scale thus depends on the value of f rather than on the individual values of a and b.

§ 4. The Linear Zone Plate

It is possible to divide up a cylindrical wave-front into a set of linear half-period zones, and thus get a linear zone plate analogous to a cylindrical lens.

There are, in practice, three ways of doing this wave-front division. In the first, the wavefront is left open from the centre of the wave to



CORNU'S SPIRAL.

Fig. 3.—Cornu's spiral for type I linear zone plate

points where the disturbance is $\frac{1}{2}\pi$ out of phase (fig. 3). This occurs when the tangent to the Cornu spiral first becomes vertical, and this point occurs when $v=1$. The next zone, which is blackened, runs from here to the next position with a vertical tangent, at a phase angle of $\frac{3}{2}\pi$ and a v value of $\sqrt{3}$. As the angle ψ of the tangent is always given by $\psi = \frac{1}{2}\pi v^2$ we get zone boundaries at v values given by:

$$v^2 = 1, 3, 5, 7, 9, \text{etc.}$$

With this system of zone boundaries, we get, for one-half of the zone plate, a vector from the origin of the spiral to the point where $v=1$, which lies at an angle $\tan^{-1}\dfrac{0{\cdot}4383}{0{\cdot}7799}$ to the x axis, and then a set of substantially horizontal vectors arising from the outer zones. The lengths of these vectors can be obtained approximately from the radius of curvature formula of the spiral:

$$\rho=\frac{1}{\pi v}.$$

For example, the open zone from $v^2=3$ to $v^2=5$ can be regarded as having an average v^2 value of 4, giving $v=2$ and $\rho=1/\pi\sqrt{4}$. The length of the $3\rightarrow5$ vector will thus be $2\rho=2/\pi\sqrt{4}$. Similarly the length of the $7\rightarrow9$ vector will be $2\rho=2/\pi\sqrt{8}$. Thus the sum of the outer zones:

$$S_{\text{outer}}=\frac{2}{\pi}\left[\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{8}}+\frac{1}{\sqrt{12}}+\ldots+\frac{1}{\sqrt{(4m)}}+\ldots\right]$$

or

$$S_{\text{outer}}=\frac{1}{\pi}\left[\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\right].$$

But the above sum is divergent. Thus, in the absence of an obliquity factor, these horizontal vectors add up to an indefinitely large amount, and ultimately swamp the vector from the inner zone. The system thus gives a maximum on the axis at the chosen point.

When we examine the possible higher orders of this system, we find that the even orders vanish, because the outer zones contribute regions which are substantially closed circles on the Cornu spiral, leaving the initial vector of length similar to that of the unobstructed path. Odd orders will be present, as the open areas then correspond to an odd number of $\frac{1}{2}$ turns in the spiral giving a divergent series of substantially horizontal vectors.

The second type of zone plate obtainable is that which results by considering the intersection of the Cornu spiral with the line $x=y$. This was the system originally chosen for practical experiments, as all the vectors lie along $x=y$ in the focal position. Also, it gives a maximum value for the initial vector. The diagram for this type is shown in fig. 4. A finite zone-plate of this type is shown in fig. 5.

In order to get the "v" values we have to solve the equation $C(v)=S(v)$ where $C(v)$ and $S(v)$ are the Fresnel integrals. This can be done by inspection of the tables, but approximate roots can be readily obtained. For this, we assume that Cornu's spiral always crosses the line $x=y$ at right angles. Although not strictly accurate for the first turn or two, the approximation becomes rapidly better as v increases.

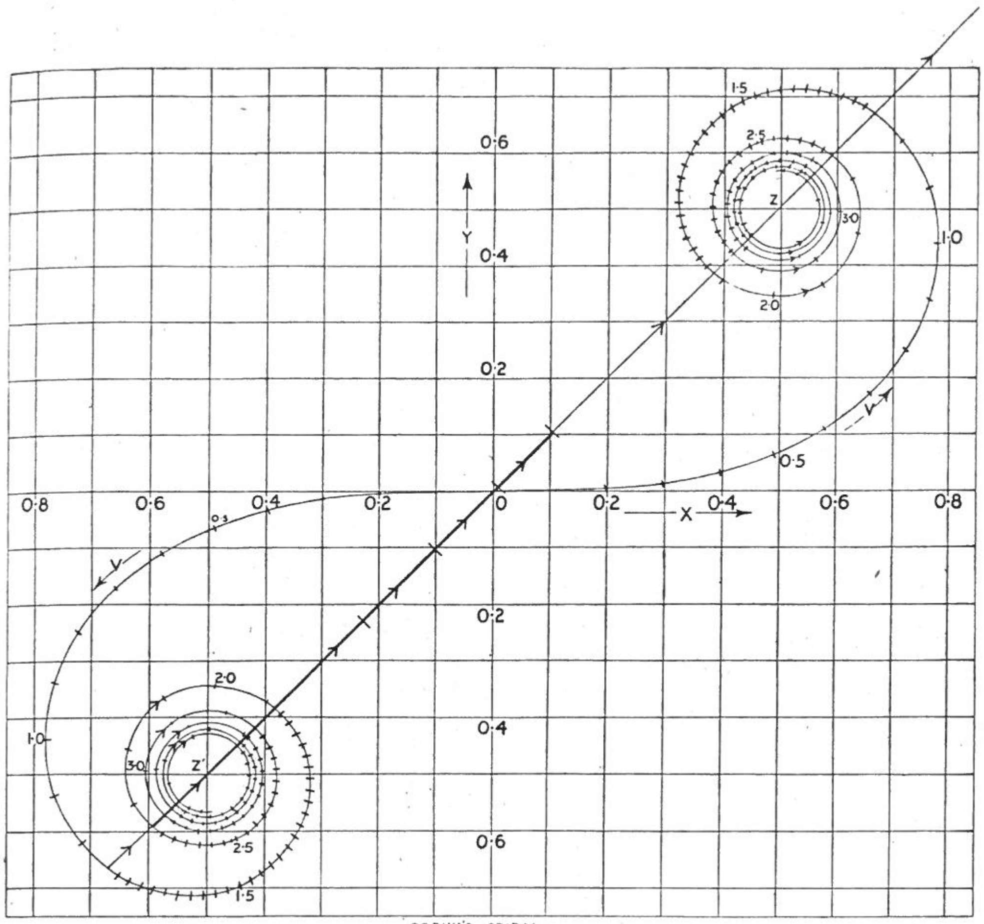


FIG. 4.—Cornu's spiral for type II linear zone plate

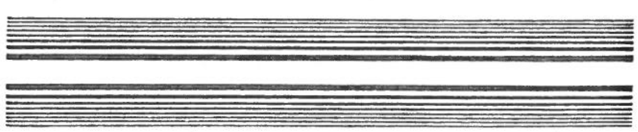


FIG. 5.—Type II linear zone plate

Since the slope of $x = y$ is $+1$ this means we have to find the points on Cornu's spiral where the slope is -1.

If $\tan \psi = -1$, then $\psi = \frac{3}{4}\pi \pm n\pi$ and we also have $\psi = \frac{1}{2}\pi v^2$. Hence the solution of the equation is:

$$v^2 = 2n - \tfrac{1}{2}$$

or

$$v^2 = 1\tfrac{1}{2},\ 3\tfrac{1}{2},\ 5\tfrac{1}{2}, \ldots, \text{etc.}$$

We now have the range 0 to $v^2 = 1\frac{1}{2}$ open, close $v^2 = 1\frac{1}{2}$ to $v^2 = 3\frac{1}{2}$ and open $v^2 = 3\frac{1}{2}$ to $v^2 = 5\frac{1}{2}$.
This latter gives a vector along $x = y$ to add to the first vector. Taking $4\frac{1}{2}$ as the average value of v^2, we get its length to be $2\rho = 2\sqrt{2}/\pi\sqrt{9}$.

The outer vectors then become:

$$\frac{2\sqrt{2}}{\pi}\left[\frac{1}{\sqrt{9}} + \frac{1}{\sqrt{17}} + \frac{1}{\sqrt{25}} + \dots + \frac{1}{\sqrt{(8m+1)}} + \dots\right].$$

This, again, is a divergent series indicating a bright line corresponding to the first order.

The high order effects with this grating differ a little from those of the first type. Even orders are missing, as with the other types and for the same reason. The outer zones make little or no contribution, being virtually closed circles.

The first order which produces vectors along the line $x = y$ is the fifth order. The third order produces vectors from the outer zones which lie substantially at right angles to the line $x = y$. As this series is also divergent, we must suppose that this vector will swamp the original vector, and give a bright line with a phase $\frac{1}{2}\pi$ different from the first order.

The fifth order series is opposed to the initial vector, and must swamp it, giving a disturbance π out of phase with the first order disturbance. With a finite zone plate, of the right number of terms, the 5th order system may give darkness, the finite series from the outer zones being equal and opposite to the initial vector.

Similarly the 7th order gives a system of vectors perpendicular to $x = y$ and $\frac{3}{2}\pi$ out of phase with the initial vector. The 9th order is the first where the outer vectors are parallel to the initial vector and in the same sense.

We have shown that the sum of the outer vectors for the first order is

$$\frac{2\sqrt{2}}{\pi}\left[\frac{1}{\sqrt{9}} + \frac{1}{\sqrt{17}} + \frac{1}{\sqrt{25}} + \dots + \frac{1}{\sqrt{(8m+1)}} + \dots\right].$$

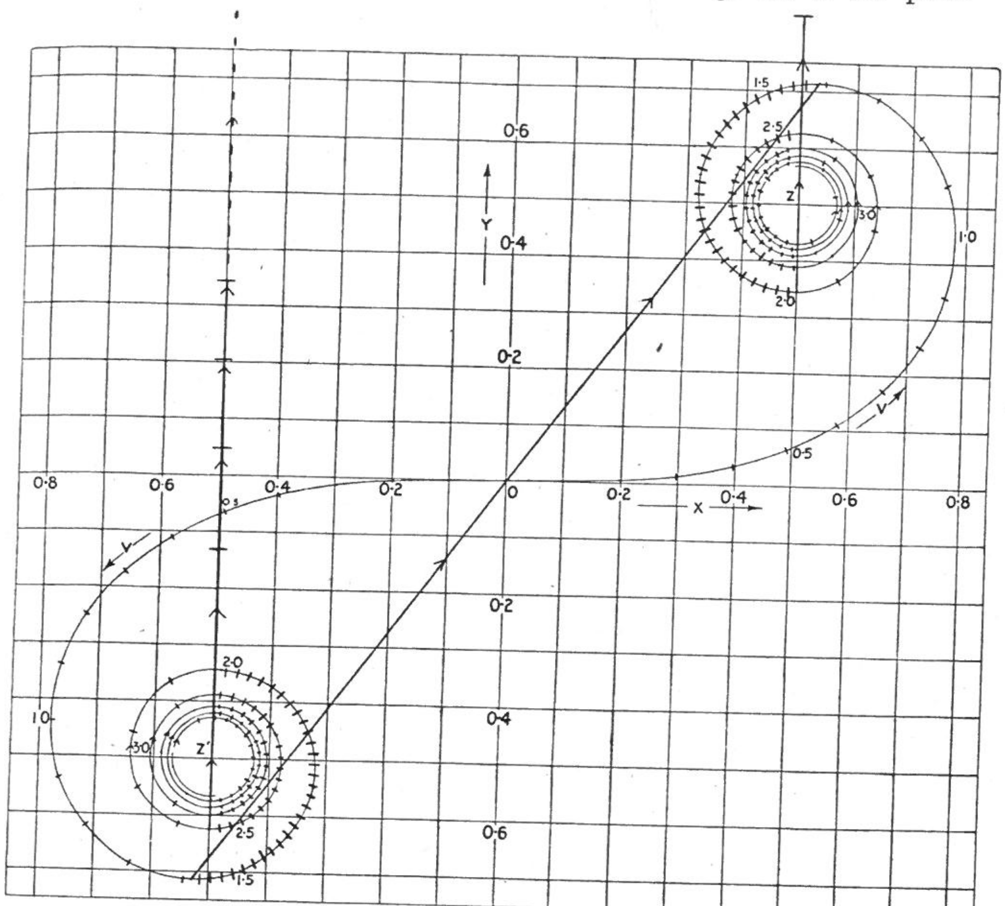
Similarly, by considering the average radius of the approximately circular arcs in the 5th order, we get a sum:

$$\frac{4}{\pi\sqrt{10}}\left[\frac{1}{\sqrt{9}} + \frac{1}{\sqrt{17}} + \frac{1}{\sqrt{25}} + \dots + \frac{1}{\sqrt{(8m+1)}} + \dots\right].$$

The ratio of these two vectors is $1 : \frac{1}{\sqrt{5}}$. In general the nth order will give rise to an outer vector sum of magnitude $\frac{1}{\sqrt{n}}$ of that of the first order.

We thus see that the *intensities* fall off as the order, and become low at high orders.

The third type of zone plate has an initial zone going a little beyond the maximum initial vector from the origin to the first intersection with $x = y$. Here we allow the 1st or central zone to go out to the point



CORNU'S SPIRAL

Fig. 6.—Cornu's spiral for type III linear zone plate

where the phase is π, *i.e.*, where the Cornu's spiral is next *horizontal* (fig. 6). This occurs, of course, where $\frac{1}{2}\pi v^2 = \pi$ and gives $v^2 = 2$ as the edge of the zone. Next blot out the zone from $v^2 = 2$ to $v^2 = 4$ and leave open $v^2 = 4$ to $v^2 = 6$. This latter gives a vertical vector directed upwards. We thus get a series, with zone edges at $v^2 = 2, 4, 6, 8 \ldots$ etc. Dividing throughout by 2, the ratio of these distances (in v) is as the square roots of the natural numbers, as in the circular zone plates. Once again, the series is divergent, indicating a swamping of the initial vector.

Even orders once again disappear and odd orders make their appearance, but without alternations in phase.

The use of a linear zone plate with a finite number of terms may require special consideration. In fact, experimental work has shown that with a suitable finite number of zones, black lines can be obtained instead of white ones. The detailed consideration of these problems is deferred until a later paper.

§ 5. Apparatus

In the early work an ordinary Osira lamp was used, as light source, with the glass bulb removed, and the whole in a box to protect the operator from U.V. light. An image of the arc was focused by a 16 mm. microscope objective on to a small hole 10–12μ diameter of approximately circular outline made by Boy's method (Strong, 1938). This is not an entirely satisfactory procedure, because it is difficult to produce a very fine hole, but it does very greatly reduce the number of spurious fringes otherwise produced by dust in the system. The main difficulty lay, however, in the moderate intrinsic brilliance of the Osira lamp.

Later work was therefore carried out with one of the recently developed high-pressure mercury lamps, running at 250 watts, and developing the greater part of the light in a region between the electrodes of 3 to 4 mm. diameter. The plasma in the Osira filled a tube 1 by 3 cm. This compact source was run in the region of a disk carrying a number of pin-holes of various diameters, from $20 \longrightarrow 1{,}000\mu$, which disk could be rotated from outside the lamphouse to bring each hole into play in turn as required. The pin-holes allow a cone of light to pass, and this falls on to one side of a microscope objective, either directly or after passing through a colour filter mounted on another disk. The distance of hole to objective is the "working distance" of the objective, and the side of the lamp-house is made to carry the objective. This arrangement forms a reduced image of the hole, outside the lamp-house, in a convenient position for use (fig. 7).

The "optical bench" consists of a wooden shelf with two metre sticks screwed to it to give an edge against which other units could be slid. These consisted of (i) an object holder, (ii) a plate holder, (iii) a hologram holder, (iv) a 7 in. Aero-Ektar lens, (v) an eyepiece, and occasionally (vi) a small lensless "camera". A night microscope (Rogers, 1948) was also available but was not greatly used when the new arc was put into operation (fig. 8).

(i) The object holder simply consisted of a jig to hold 1 to 4 thicknesses of $\frac{1}{4}$-in. patent plate glass, on which photographic objects were mounted.

(ii) The plate holder was in fact quite a complex device which was designed to allow a printing frame, loaded in the dark-room with a plate, to be put in an accurately pre-determined position. This consisted of a flat board resting on the shelf and a vertical front with a hole in it. The frame was held against this front with springs. Provision was made for the insertion of a filter or compensating plate before the frame, if neces-

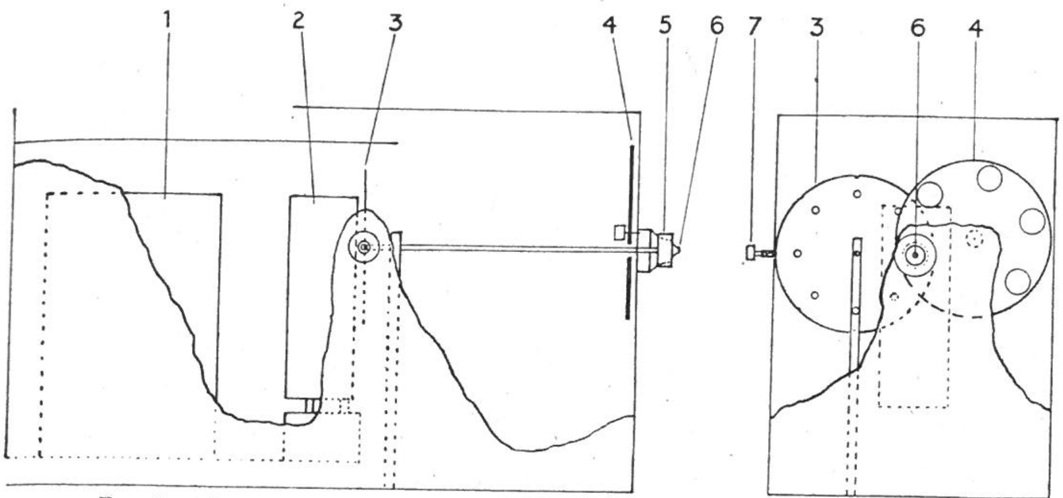


FIG. 7.—Diagram of lamphouse. (1) Choke. (2) Box-type arc. (3) Hole disk. (4) Filter disk. (5) Hole-changing knob. (6) Objective. (7) Device for locating hole disk

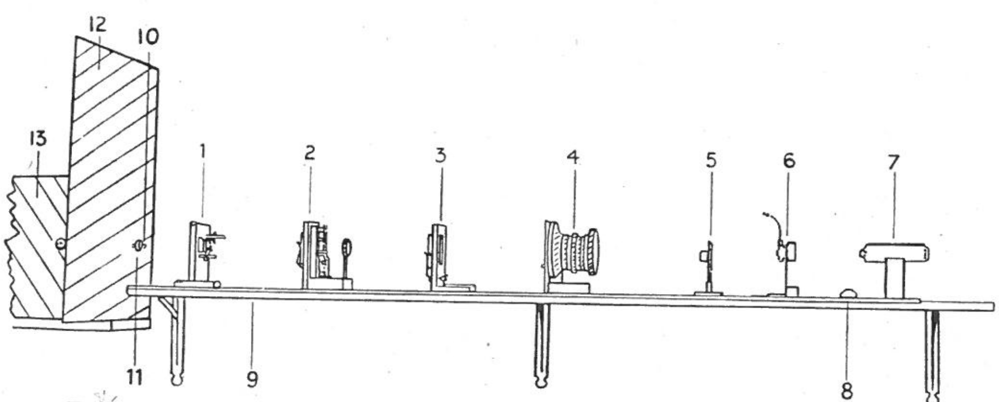


FIG. 8.—Etch Bleach drawing of the apparatus, based on a photograph. (1) Object holder. (2) Plate holder. (3) Hologram holder. (4) Aero-Ektar lens. (5) eyepiece. (6) Lensless camera. (7) Night microscope. (8) Spare objective for night microscope. (9) Shelf. (10) Microscope objective. (11) Knob for changing holes. (12) Screen. (13) Lamphouse.

sary. A medium-focus convex lens, placed behind the frame, was focussed on the plane holding the plate. This thus enabled the whole carriage to be sited in the correct position, the frame being away, by using this lens as a low-powered eyepiece.

(iii) The hologram holder was a simplified version of the above, without lens, designed to hold a hologram against a vertical surface with a hole in it, for convenience in reconstruction. The next two items (iv) and (v) are self-explanatory.

(vi) The small lensless camera consists of a shutter with a circular chamber behind. The back surface of this is a cap, with arrangements for holding a plate the size of a postage stamp. It was originally designed for work with an experimental Bragg "X-ray Microscope", but was useful for photographing the reconstructed images of holograms, when an auxiliary lens was employed.

For normal purposes a run of 2 metres was sufficient, but for certain experiments use was made of a shelf at the same level in an adjacent room, giving in all a possible run of 13 metres.

§ 6. Technique

The essence of diffraction microscopy is the production and photography of Fresnel Diffraction patterns. For this purpose monochromatic effects are necessary. The mercury arc gives a number of components in the visible, the most important of which are those at 4358, 5461 and the doublet at 5770—5791 Å. The lines at 4047 and 4078 are, fortunately, comparatively weak, and the two red lines at 6152 and 6232, while useable, are not very strong. There is also a strong U.V. line at 3650 Å. which can fortunately be suppressed easily by any standard U.V. cutting filter.

To obtain a photograph in monochromatic light, it is unnecessary to filter the radiation until it contains only one component. Any number of wavelengths may be allowed to fall on the plate, as long as the plate is sensitive to only one of them. Using a well-known series of filters with sharp short-wave cuts, in combination with a plate of suitable sensitivity, each of these lines can be isolated.

In practice it is found that the following combinations are effective: Ilford "Q" filter to cut the U.V. and partially suppress 4047 and 4078, together with an "ordinary" plate will record only 4358. An ordinary plate contains a pure silver-bromide emulsion without any sensitizing dye. For recording the 5461 it is convenient to use the Wratten 77 or 77A, which not only cuts the blue but suppresses the yellow. If this is used with any orthochromatic plate, the red passed by these filters is quite unimportant. The yellow can be isolated with the Wratten 22 filter. The usual type of orthochromatic emulsion is a little slow to this radiation, but a panchromatic emulsion might record some red. Finally, the red lines can be isolated by an Ilford Narrow Cut Red or a Wratten 26, and recorded on a panchromatic plate. This procedure is slow, and was only used in wavelength dependence tests. Consequently the blue and green lines were most frequently used.

The objects were normally small scale reproductions from black and white line drawings, though one or two attempts to reproduce continuous tone originals have been made. The normal procedure was to remove the shutter from the lensless camera and replace it with a microscope objective screwed into a suitable mount. By successive exposures, it was possible to focus this camera, and when this was done a series of objects would be prepared before disturbing the adjustment. Although technically the objective should have been used at its "working distance" of 160 mm. it was found that quite satisfactory results could be obtained at 1 or even 2 metres. Maximum Resolution plates were used throughout, and the progress of the work was followed under a microscope. The objects, when dry, were cemented with D.P.X., a standard microscopical mounting agent, on to a $\frac{1}{4}$-in. patent plate glass.

The initial diffraction patterns were formed by allowing light to diverge from the star image of the source through the object, and thus on to the plate. The effective positions of source and object were ascertained by using the Aero-Ektar to form images of each, which were located by the eyepiece. The actual positions were then obtained by Newton's formula. At a later stage these observations were correlated with the index positions of the holders, and a "zero error" established for each.

Owing to the readiness with which quarter plates can be obtained, these were taken as a standard plate size. A $2\frac{1}{2}$-in. square aperture at one end of the plate received the diffraction pattern, through a hole in the mask, and an area $2\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. at the other end was subsequently exposed to the same radiation through a Kodak No. 1 step wedge. This consists of a set of densities from 0 to 1·5 in steps of 0·15 approximately. It also carries three coloured patches, red, green and blue, originally intended for the identification of three-colour separation negatives. To modify it to identify the four principal mercury lines, a small patch of Wratten 22 was added. This step wedge enables a measure of control to be exercised over the processing, and also provides an index of the radiation by the use of the patches.

The plate is loaded into a printing frame, carrying the mask, in the dark-room and wrapped in black paper for transfer to the optical bench. It is located in the holder, and exposure is effected by removing the black paper for the required time. The step wedge is exposed immediately afterwards.

Development is carried out in an ordinary M.Q. developer to a medium contrast, depending on the emulsion used. An estimate of the contrast is made by eye, and the printing technique varied accordingly. Low contrast negatives are printed on process plates (by contact) and high contrast negatives on "Ilford Ordinary" which gives a softer result. In accordance with the recommendation of Gabor an over-all contrast of about two is aimed at, but it is not found necessary to achieve this with any very great precision. A value a little less, if anything, is favoured, as higher values produce a characteristic effect of "burning out" dark lines, to give an artificially light centre.

The positive print is mounted with D.P.X. on a $\frac{1}{4}$-in. plate, as for the object, and placed in the hologram holder. The object is removed, and the reconstructed image is photographed directly with the plate holder, or indirectly with the Areo-Ektar and lensless camera. The former is generally preferred. Possible arrangements are shown in fig. 9.

§ 7. Existence of a Focus

As indicated in the theoretical sections, significance is attached to the parameter, f, and experiments were directed to the elucidation and verification of this idea and its consequences.

With the H.P. Lamp, the major part of the work was done with star images. In particular, a hologram negative (H 138) was taken in blue light (4358) of an object which was a photographic replica (by contact printing) of a micrometer eyepiece scale 1 cm. long divided into 100 parts.

A positive print of this (H 140) was mounted and subjected to careful study in Hg green light. In particular the source-hologram distance was varied and the image located. There are two ways of doing this, both using an auxiliary lens. In the first place the auxiliary lens is used to form a real image of both source and subsidiary images. These may be located by an eyepiece. The method of no-parallax is *not* available since all the light passes through a single point region, and no differences of viewpoint are available. Location must thus depend on an impression of sharpness by no means easy to ascertain with a highly coherent illuminating system. It was thus felt that a single eyepiece determination would be of doubtful value, and in practice a series of at least six observations, with the auxiliary lens moved between, were taken as a single "location run" and the image position calculated from each observation. The mean of these observations was taken as the position of the image.

This procedure, though tolerably accurate, was tedious and fatiguing. The following alternative was developed. The auxiliary lens was used to produce a real image of source and reconstructed images, and also of the object, left in the path between the source and hologram. Normally either object or hologram is inverted to avoid overlapping. If, now, the object is moved back and forth, it may be brought into coincidence with one (but only one) of the reconstructed images, as judged by the fact that object and reconstructed image are equally sharp as seen in the eyepiece. A photograph of this composite field is shown in fig. 10, Pl. II.

In this way it was only necessary to carry out one location run, at the beginning of the series, and get all other positions by observing the

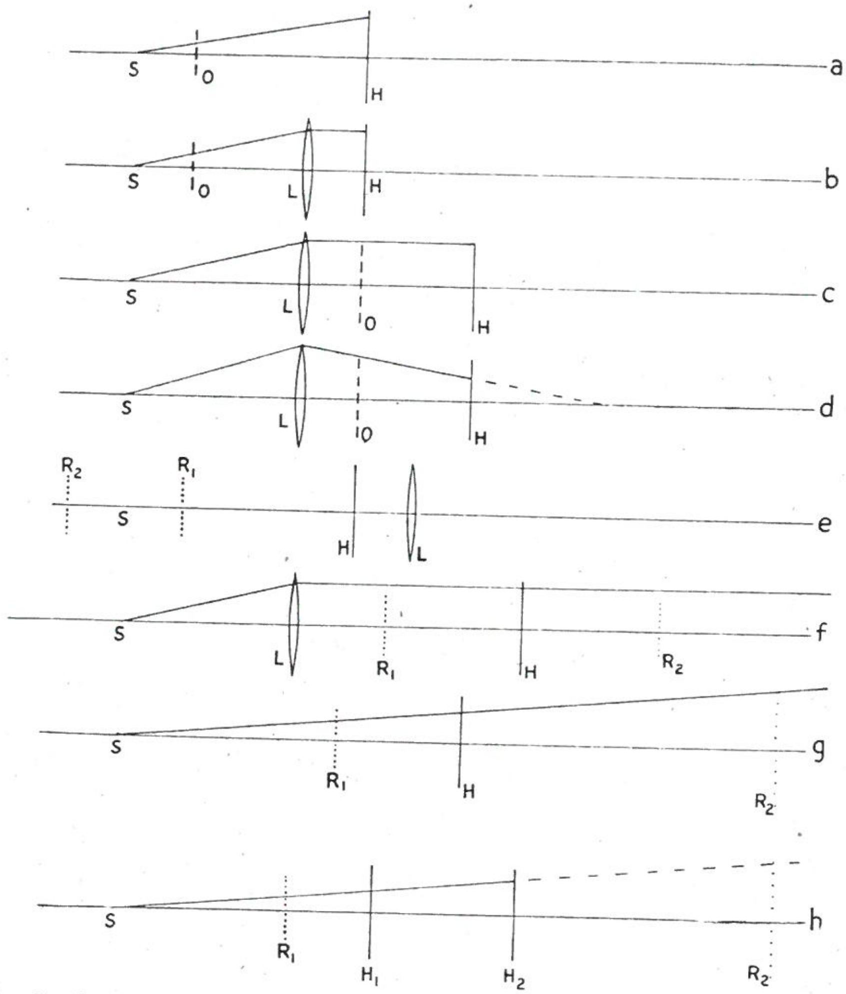


FIG. 9.—Some suggested systems. - - - - object, ——— hologram, ... reconstructed image. (*a*) Direct divergence. (*b*) Indirect divergence. (*c*) Parallel light. (*d*) Convergent light. (*e*) Gabor's original reconstructing system. (*f*) Reconstruction in parallel light. (*g*) Lensless reconstruction in divergent light. (*h*) Set-up for hologram of a hologram (H 157/9)

shifts of arbitrary indices fixed to the object and hologram carriages. At the end of the operation, a new location run was performed as a check.

Furthermore, since the object was a scale, and the hologram was produced from it and gives a scale on reconstruction, the effective over-all magnification of the operation can be determined at each stage by the direct comparison of the two scales, real and reconstructed, a process which gives valuable additional information as to the nature of the operation.

In this operation a slight difficulty arises in locating object and hologram from the fact that they are mounted between $\frac{1}{4}$-in. glass plates, and hence their optical position differs from their physical position. The method chosen is to locate them using the auxiliary lens, and to calculate their distance from a known focal point of this lens, using Newton's formula, on the assumption that the medium is air. This automatically gives their effective optical position. There is, however, some slight suggestion that the effective optical position varies slightly with the angle of the cone accepted by the lens.

Table I gives the index readings and located positions of source, scale and hologram at the beginning and end of the run.

TABLE I

	Source	Scale		Hologram		Zero Errors	
	Located	Index	Located	Index	Located	Scale	Hologram
Beginning	95·75 cm.	94·0	94·10	87·88	85·34	·10	−2·54
End	95·64 ,,	91·0	91·07	79·85	77·08	·07	−2·77

It is not altogether clear whether the change is due to errors in location, or whether they arise from a slight drift due to change in the effective optical position. Calculations have been made on both bases, those on the second assumption being given in Table II.

Positions are estimated to 1/10 mm., even when location is less accurate, to ensure that errors arising from the use of rounded figures in calculation shall be smaller than those due to the observations. It will be seen that both the focal length (or the power $1/f$) and the expression $Mu/(u-v)$, are sensibly constant. Experience shows that errors of 2 mm. in setting the hologram can easily occur, and hence an accuracy better than ± 5 per cent. cannot be guaranteed.

The constancy of the power was expected on theoretical grounds. The significance of the expression $Mu/(u-v)$ now requires exploration. Gabor has shown that, in the production of the hologram, the coarse structure, which is reproduced directly like a shadow, is magnified by the usual projective law. A similar law is expected to hold for the reconstruction of the image. Since the coarse structure must act as a species of "framework", it is legitimate to expect that the fine structure, when ultimately reconstituted, will also be found to obey the same law. This has been verified by experiment.

The over-all magnification M of the whole process, as observed by direct comparison of the two scales, is recorded in the 11th column. Now the projective magnification of the 2nd or reconstructive stage will be less than unity, and is given by $(u-v)/u$ in our notation. The reconstructed scale is smaller than the coarse framework recorded on the hologram. Since we know the observed over-all magnification, and also the theoretical stage 2 magnification (column 13), it is legitimate to divide the first by the second to get the effective magnification at the first stage. That is, the magnification between the original object and the coarse framework on the hologram. Since this is fixed once and for all by the initial exposure conditions, we anticipate that $Mu/(u-v)$, its measure, will also be constant, and the constancy of this expression constitutes the verification of the theory.

It is also found on reference to the conditions under which H 138 was taken, that the source was at 95·84, the scale at $93{\cdot}04_5$ and the plate at $80{\cdot}0_0$. From this we deduce a projective magnification of $\frac{15{\cdot}84}{2{\cdot}79_5}=5{\cdot}65$ in excellent agreement with the value of Table II (last column). In fact, allowing for the probable errors of location, a variation of 5 per cent. would not have been excessive.

Now we note that H 138 was taken in blue light, but the reconstructions of Table II were made in green light. We see from this that the coarse structure is magnified in a purely projective way and the resultant magnification can be calculated on this basis from source, object, hologram, and reconstructed image positions without *direct* reference to the wavelengths involved. Of course, the wavelength used has an indirect influence on the position of the reconstructed image, by its influence on the effective hologram focal length. Moreover, we are here concerned with the linear magnification: we are not concerned with the resolving power and some of the magnification may be "empty". The resolving power is very closely related to the wavelength.

The data for the conditions under which H 138 were taken lead to a calculated hologram power of 1·36 dioptres in blue light. This, of course, varies with wavelength, and so in green light we expect a power of $1{\cdot}36\times\frac{5461}{4358}=1{\cdot}704$ dioptres. The errors in the positions of source and scale as located prior to taking H 138 could produce a variation of $\pm$ 5 per cent. in the power, which is sufficient to bridge the gap between this and the value in Table II (Column 10).

§ 8. Wavelength Variations

The above results, while not inconsistent with the theory that the power varies directly as the wavelength, cannot be regarded as a particularly convincing demonstration of the fact. To investigate this phenomenon, the following experiment was performed.

Two holograms were to be taken with the same source, object and plate positions, but in different wavelengths. Bromide enlargements were to be made from each negative under fixed conditions of magnification. These enlargements were then to be cut along a line, and a "top" of one mounted above a "bottom" of the other. The enlargements were then photographed down to approximately the original size and a positive composite hologram was prepared from them.

Two attempts have been made along these lines. The first was an early attempt with the Osira. Only two wavelengths with this were sufficiently intense for use in this experiment, the blue and the green. They do not have a very large ratio (1·25:1) and hence were not very promising. H 63 was given an hour's exposure to green light (being an orthochromatic plate), while H 65 was the same object, in blue light using a fast ordinary plate for about 10 minutes. The composite hologram produced from this was called H 68, and was studied intensively in green light. The upper half of the hologram was from H 65, and the lower from H 63, the object being a reduced copy of a newspaper cutting with a chess problem.

The second experiment was done with the H.P. arc. The set-up was maintained accurately by clamping down the new object holder and plate holder to the bench. A series of photos was taken in blue and in red light, thereby getting a more favourable wavelength ratio of $\sim 1{\cdot}42$. Two were selected as being of comparable density and contrast, as judged from the hologram and step wedge respectively, so as to get bromide enlargements more closely matched than before. In the end H 186 and H 187 were chosen.

H 186 was 1 hour's exposure in red light to a fast panchromatic plate (P 1500), the object being a microscope eyepiece scale. H 187 was 2 minutes' exposure on a slow ordinary plate (Ilford Ordinary). After enlarging, the holograms were trimmed to show the scales and a small surround, and were mounted with the scales opposite one another, one being inverted for the purpose. An enlargement to the same degree was made of the step wedges, which were carefully matched and included on the hologram to enable the over-all contrast to be checked. On re-photographing the resultant positive was numbered H 193. In this case great care was taken to get the final hologram the same size as the originals, the point being checked on the focusing screen with one of the original negatives. This is to avoid any complications with the scale factor.

A run was not performed with H 193, but instead observations were made in green light rendered parallel by the auxiliary lens. In this case the focal length can be determined directly as the distance the eye-

TABLE II.—*Experiments in Diffraction Microscopy*

(Mercury green light λ = 5461 Å.)

Source	Scale Index	Scale Position	Hologram Index	Hologram Position	(cm.) "u"	(cm.) "v"	(Dioptres) 1/u	(Dioptres) 1/v	(Dioptres) 1/f	Mag. overall observed	u − v cm.	$\frac{u-v}{u}$	$\frac{Mu}{u-v}$
95·75	94·0	94·10	87·88	85·34	10·41	8·76	9·607	11·416	1·809	0·903	1·65	·1586	5·695
95·74	93·7	93·80	86·95	84·39	11·35	9·41	8·811	10·627	1·816	0·970	1·94	·1710	5·670
95·73	93·4	93·50	85·95	83·37	12·36	10·13	8·091	9·872	1·781	1·007	2·23	·1804	5·570
95·72	93·1	93·19	85·05	82·44	13·28	10·75	7·530	9·303	1·773	1·082	2·53	·1905	5·665
95·71	92·8	92·89	84·60	81·97	13·74	10·92	7·278	9·157	1·879	1·150	2·82	·2052	5·610
95·69	92·5	92·59	83·70	81·05	14·64	11·54	6·830	8·665	1·835	1·188	3·10	·2117	5·615
95·68	92·2	92·28	82·30	79·62	16·06	12·66	6·227	7·899	1·672	1·210	3·40	·2119	5·715
95·67	91·9	91·98	82·00	79·30	16·37	12·68	6·109	7·886	1·777	1·250	3·69	·2256	5·540
95·66	91·6	91·68	81·20	78·48	17·18	13·20	5·821	7·576	1·755	1·299	3·98	·2317	5·605
95·65	91·3	91·37	80·30	77·55	18·10	13·82	5·525	7·236	1·711	1·321	4·28	·2366	5·585
95·64	91·0	91·07	79·85	77·08	18·56	13·99	5·388	7·148	1·760	1·356	4·57	·2465	5·505
									Av. 1·779				Av. 5·616

TABLE III.—*H* 68 *Hg Green Light. Examination of "Symmetrical" Images*

Hologram "u"	1/u	U_1-Holog. "v_1"	$1/v_1$	L_1-Holog. "v_2"	$1/v_2$	$1/f_1$	Average	$1/f_2$	Average
11·19	8·94	14·38	6·96	13·46	7·43	1·98		1·51	
12·72	7·87	17·08	5·85	15·71	6·37	2·02		1·50	
13·56	7·38	18·59	5·38	16·99	5·88	2·00		1·50	
14·75	6·78	20·62	4·85	18·92	5·29	1·93	$1·99_6$	1·49	1·51
15·54	6·43	22·52	4·44	20·18	4·96	1·99		1·47	
16·64	6·01	25·26	3·96	22·69	4·41	2·05		1·60	
		L_2-Holog. "v_3"	$1/v_3$	U_2-Holog. "v_4"	$1/v_4$	$1/f_2$		$1/f_1$	
11·19	8·94	9·64	10·38	8·93	11·19	1·44		2·25	
12·72	7·87	10·84	9·22	10·20	9·80	1·35		1·93	
13·56	7·38	11·40	8·77	10·68	9·37	1·39		1·99	
14·75	6·78	12·33	8·10	11·41	8·77	1·32	1·39	1·99	2·01
15·54	6·43	12·72	7·86	11·91	8·40	1·43		1·97	
16·64	6·01	13·59	7·41	12·60	7·93	1·40		1·92	

piece moves between focusing on the hologram and on the reconstructed image. A number of settings were made, which showed variations of $\pm$ 3 mm. for the shorter focus, and $\pm$ 6 mm. for the longer focus. Both are thus within $\pm 2\frac{1}{2}$ per cent. The focal lengths were 20·31 cm. and 28·91 cm., corresponding to powers of 4·93 and 3·46 dioptres respectively. The ratio of these powers is 1·425, in excellent agreement with the ratio of 1·42 of the wavelengths.

A further check is obtained from the taking set-up. The spacing of source, scale, and plate leads to a power of 3·87 dioptres; this being 3·87 dioptres in blue for H 187 and in red for H 186. When these powers are converted from blue and red into green they come out at $4{\cdot}84_5$ and 3·41 respectively, which gives good agreement with the observed powers.

It seems fair to accept a direct relation between power and wavelength, as indicated by the theory. This being so, we can go a stage further, and get a more fundamental constant of the hologram. Hitherto we have specified the power of a given hologram in a given wavelength, but since these are proportional to one another, we can now specify the power per unit wavelength. In order to get numerically convenient quantities from ordinary holograms it is suggested that the power per unit wavelength be normally specified in dioptres per micron. We get for H 187, then, a power per unit wavelength of 8·88 dioptres/micron and for H 186 of 6·25 dioptres/micron. It will be observed that this unit has the dimensions L^{-2} and a magnitude of 1/sq. mm. It is called the power-rate.

For comparison it might be pointed out that for an electron system with a source-object distance of 0·1 mm. and a source-plate distance of 1 metre, using electrons of 0·05 Å. wavelength, the power-rate will be about 20 dioptres/micron. Such a hologram will thus be comparable with the optical ones here considered.

§ 9. The Scale Factor

It was early appreciated that the scale on which the hologram positive actually used is reproduced from the negative as originally taken, would be important and would be worth study as throwing light on the theory of the process. While, therefore, contact printing was normally used for reproduction, an enlarger was also employed in some cases.

In the first place a couple of pairs of holograms were prepared: each containing (*a*) a contact print, and (*b*) a *reduction* in the enlarger. The latter was preferred to an enlargement, because it gave a hologram easier to handle. This arises from the fact that such a hologram has an increased power-rate.

In one case the pair was examined in parallel light, and the focal lengths determined as for the composite hologram, H 193. We got, for the normal size, $f = 21{\cdot}47 \pm {\cdot}59$ cm. and for the reduced hologram $f = 11{\cdot}35 \pm {\cdot}34$ cm. This leads to a ratio of $1{\cdot}89 \pm {\cdot}11$. Careful measurements on these holograms give the linear ratio as $1{\cdot}351 \pm {\cdot}01$. This corresponds to a $(\text{ratio})^2$ of $1{\cdot}8252 \pm {\cdot}03$. It will be seen that the law $f \propto L^2$ holds within the limits of experimental error.

The other pair was measured without an auxiliary lens, by u and v measurements over a considerable length of bench. The powers obtained were: Normal 0·55 dioptre, reduced 1·46 dioptre. Ratio 2·67. Linear reduction 1·573, which squared gives 2·48.

As a further check, one particular hologram was enlarged. This was H 175, taken in parallel light, with the unusually high power-rate of 17·3 dioptres/micron. This is the only hologram which we felt strong enough to stand "dilution". It was subjected to a linear magnification of $2{\cdot}76 \pm {\cdot}07$ and focal lengths were obtained as follows (in Hg green). Normal $10{\cdot}61 \pm {\cdot}2$ cm. Enlarged $78{\cdot}92 \pm 1$ cm. Ratio $7{\cdot}44 \pm {\cdot}24$. Square of linear magnification $7{\cdot}62 \pm {\cdot}35$. Here again, the agreement is satisfactory, and the fact that the linear ratio here departs substantially from unity is an additional check on the theory.

§ 10. The Two Images

It is an essential of the theory that both zone plate and hologram produce two images. So far we have only examined one image at a time. The experiments first described located the image produced by H 140 acting as a divergent lens. Some of the later experiments, determining the focal length directly, use the hologram as a converging lens. It remains to show that the converging and diverging powers are the same.

Reference may here be made to some work with the early composite hologram H 68 in green light. The work was all done with the old Osira and was thus not carried out under the most favourable conditions. The method of locating an image with a moveable object had not been developed (and would not have located the second image). So all data were obtained from "Location runs". This makes the analysis very tedious, and the work has not been repeated.

Measurements were made to locate (i) the upper image sharp, (ii) the lower image sharp, (iii) the source, (iv) the "symmetrical" lower image sharp, and (v) the "symmetrical" upper image sharp. The hologram itself was located separately.

It at once became apparent that the "symmetrical" images were not symmetrical. Gabor's deduction of symmetrical images rests on the assumption that the image source distance is very small compared with the source-hologram distance, *i.e.*, that f is very large. In this case, indeed, the lens formula does give symmetrical images.

The results are summarized in Table III.

An examination of Table III shows at once that the results obtained from the upper part of the hologram (U_1 and U_2) are as satisfactory as can be desired. This part of the hologram contained resolvable writing, and was processed to very nearly the ideal contrast. Hence this image was easy to locate. The lower part, L_1 and L_2 does not give the same sort of agreement. Here the hologram contains no resolvable writing, and is of lower contrast and hence the locations were more difficult. It is not felt that the divergence is beyond the limits of experimental error.

Working from the estimated power on taking, and assuming the zone-plate law, we get theoretical powers of 1·67 and 2·01 dioptres for the two cases. But owing to the practical difficulties of being sure of the correct positions before the taking device was made, these figures must be taken as indicating an order of magnitude only.

We see, therefore, that the theoretical expressions derived from the simple theory of the zone plate are found experimentally to hold for holograms in general, within the limits of experimental error. We therefore feel justified in concluding that the hologram is a generalized zone plate and in particular we associate it with the following properties :

(1) A given Fresnel diffraction pattern, produced in coherent monochromatic light of wavelength λ proceeding through a point may be associated with a focal length f, being the focal length of that lens which, placed in the plane of the pattern, images the source (or virtual source) in the plane of the object producing the pattern.

(2) If a photographic reproduction of the pattern is now placed in a coherent beam of light of wavelength λ it will form two "images" of the source, resembling the object, as though it were a lens of focus $\pm f$.

(3) If it be placed in a beam of wavelength λ' it will act as if it had a new focal length $\pm f'$ where $f'\lambda' = f\lambda =$ a constant of the pattern. In particular $1/f\lambda$ is called the power-rate.

(4) If the scale of the pattern be altered by a linear factor L, it will be found that the constant factor $f\lambda$ is altered by a factor L^2.

(5) The effective magnification between the original object and its reconstruction can be calculated by purely projective considerations, from the original source through the object to the hologram, and then from the hologram through the reconstruction to the reproducing source.

The projective law follows from the "coarse structure" relation, and means that the hologram of an object (as against a point or line) must be described by *two* parameters. One is f: this gives the size of the fringe structure round any particular point. The second is the projective magnification, this determines the separation between the *centres* of these fringe systems in terms of the separation of the two original points.

This also explains why "high order" images are not normally observed. If we reduce f to $f/3$ we will normally reduce the fine structure in one ratio and the coarse in another: one varies with f and the other with the projective magnification.

§ 11. Multiple Operations

When it has been proved that an operation can be performed once, it is frequently instructive to perform it twice. Accordingly H 140 was taken and put about a metre (103 cm.) from the source in green light. Under these conditions it would produce two images, a real image about 123 cm. further away in a direction opposite to the source, and a virtual image 36 cm. on the source side of the hologram. A plate was put up 45 cm. on the more distant side of the hologram, so that the light passed through the hologram and fell on the plate before forming the real image. The plate thus records, in effect, two images, one being 81 cm. one side of it, and the other (unformed) 78 cm. on the other side of it.

The hologram, H 159, formed from this —ve has *two* effective focal lengths, $\pm f_1$ and $\pm f_2$, and produces *four* images from a point source. It is shown in Fig. 15, Pl. II.

Consideration of the taking arrangement suggests that these two foci should be $\pm$ 51·2 cm. and $\pm$ 180·3 cm., but that neither is likely to be estimated to better than $\pm$ 5 per cent., if that. Direct measurement of the foci gives 47·1 and 168·6 cm. respectively, again to $\pm$ 5 per cent. or a little better. In view of the fact that none of these images is as easy to see as is the case with a simple hologram, this agreement may be regarded as satisfactory.

Now one method of reconstruction is virtually this: the hologram is put, say, 100 cm. from the source and forms a real image at a greater distance, say 223 cm., as above. If a plate be placed here, it will record the image and this constitutes a useful and legitimate method of reconstruction. But this reconstruction is itself a hologram of a hologram and should have two focal lengths. One will be zero: and as $+0$ is the same as -0 we have here a case of coincident roots. The hologram and image become the same thing. But there will also be a subsidiary pattern from the virtual image between the source and the original hologram. Gabor recognizes this when he says that the unwanted image interferes to a slight extent with the wanted image. But the unwanted image does more than that; it records on the reconstruction, albeit diffusely, *all the information required to reconstruct it.*

We took one of our reconstruction negatives (R7) (fig. 12, Pl. I) and printed it to form a hologram H 162. This hologram was examined in parallel light, and as well as having the trivial focal length $\pm$ 0 (giving a sharp image in its own plane) it was found to have a focal length of 141·0 cm. within the usual limits. The anticipated focal length from the details of R7's taking was 145·5 cm., in satisfactory agreement.

We also made a reconstruction (R 11) (fig. 14, Pl. I) from this hologram to provide visual proof of the existence of the secondary image associated with the original reconstruction R 7.

Of course, this can in theory be done indefinitely. R 11 is a third-order hologram, and in theory might give rise to eight images. In practice owing to the existence of several coincident roots, there will be five, one of which coincides with its own plane. No other third-order holograms have yet been produced, but it might be instructive to try and produce one from H 159 in such a way as to produce coincident roots in a place other than in its plane. This might give an extra clean image, or even give rise to "beating" effects in the plane of the third-order hologram.*

§ 12. Miscellaneous Experiments

It is inevitable in an investigation of this kind that a number of side experiments are set up to explore minor ramifications of the process. A number of these are worth putting on record.

(*a*) An early experiment was made with two objects so sandwiched in glass as to lie in different planes, and thus at different distances from source and hologram. The hologram so produced, reconstructs the two objects in different planes, with such overlapping of the out-of-focus outlines as would occur in direct viewing through an equivalent system. The phenomenon can be explained by noting that each contributes to the net effect a diffraction pattern with a characteristic (but different) focal length, and the positions of the reconstructed images can be obtained from this.

(*b*) A similar effect can be obtained by taking two holograms of different objects with different power-rates. These are then double-printed on to a single plate, *i.e.*, a given plate is exposed an appropriate time behind each negative in turn and then developed. The diffraction pattern which results differs in detail from that of case (*a*) but the visual effects which result are very similar. Once again the two images can be partially separated by their different focal lengths.

(*c*) In one case a copy negative was made from a positive (with an over-all γ from the original pattern of ~ 2) and used as a hologram. This has the same focal length as the positive, and produces a reconstructed image which is a negative. R 7 and R 8 (figs. 12 and 13, Pl. I) are reconstructions from a positive and a negative hologram (positive

* Note added 5.9.51. This has since been identified as a reconstruction of the first order hologram.

hologram shown as fig. 11, Pl. I) under identical conditions, and they are mutually super-posable to give a sensibly uniform density. The exact theory behind this is not known, as it was not expected from Gabor's analysis, but the exactitude with which it works is impressive. If the initial negative can be developed to a γ of 2 (and this will only be possible with certain emulsions) there is a possibility here of economy in running costs. For a plate or paper set up in the reconstruction plane now gives a direct + ve. As the plates used here are not all suitable for high γ working (and this has other disadvantages) the method has not been much used, but some of the electron-sensitive emulsions will be more amenable to this type of treatment.

(*d*) It follows from (*b*) and (*c*) that holograms can be subtracted. In particular, it should be possible to subtract a hologram of the background from that of background + wanted image, to obtain the latter alone. The successful accomplishment of this would require a carefully adjusted "dummy" object (plain glass of equivalent thickness) and hence it has not been attempted.

(*e*) A relief image in plain (transparent) gelatine has been prepared by the Carbro process from a hologram (H 135) of the scale. By itself, this relief image introduces excessive phase distortions into a coherent beam. But if a covering plate is mounted over it using D.P.X. cement, there remains just sufficient difference of refractive index between the gelatine and the D.P.X. to produce a delicately graded phase-contrast object. This phase contrast hologram reconstructs as before. The analogy is with a zone-plate in bichromated gelatine with a $\frac{1}{2}\lambda$ retardation in the gelatine areas. As is well-known, this zone-plate acts like a normal one, but with greater light-gathering power.

(*f*) A bromide enlargement was made from a hologram negative, and the dark fringes were inked over with Indian Ink. The bromide image was then removed with Farmer's reducer to leave the ink drawing on an otherwise white ground. This was photographed down to form a hologram (called the black-and-white hologram). The subject was the microscope eyepiece scale, and the black and white hologram, though only containing three or four fringes, gave a crude but recognizable reconstruction. The method is chiefly advantageous in allowing background dirt to be ignored. A fuller description, with photographs, is given in Rogers (1950).

(*g*) An attempt was made to produce a composite hologram containing two side-by-side images in dye, produced by dye-coupling developers. The idea was to use the whole radiation to illuminate the system and get one side of the hologram absorbing only one-wavelength, the other only another, and thus effecting reconstructions in two different wavelengths in adjacent positions. But the dyes used did not interact in the way hoped, and tended to absorb too wide a wavelength band.

(*h*) An attempt was made to produce a hologram in a beam of parallel light and this was successful. The only conditions are that the object shall be small compared with the diameter of the beam of parallel light, and that the hologram shall be recorded reasonably near to the object, so that the diffraction pattern of the object still falls wholly within the area of plate illuminated by the parallel beam. The object-plate distance is automatically the focal length of the hologram, and this is how the short-focus (high power) hologram mentioned in § 9, was obtained.

(*i*) It is worth placing on record the fact that we have, in effect, produced a hologram in convergent light. About four years ago, while doing some experiments with the honours class a number of Fresnel diffraction patterns were produced in convergent light. The light from a small source (Hg Arc with pin-hole) was made slightly convergent with a telescope objective. It was found in this way that (i) a slightly larger pattern could be condensed on to a slightly smaller photographic plate, and that (ii) a concentration of light was produced which gave a welcome reduction of exposure.*

§ 13. Methods of Reconstruction

It is convenient here to summarize the methods of producing a reconstruction from a hologram. (i) There is the original Gabor method of projecting the small reconstructed image near the original object position, on to a plate by the use of an auxiliary lens. (ii) The light from the point source may be rendered parallel by an auxiliary lens, and the image observed or photographed in the focal plane of the hologram. (iii) The auxiliary lens may be dispensed with, and the divergent beam passed through the hologram, which must be further from the source than its own focal length, and the resultant real image observed or photographed.

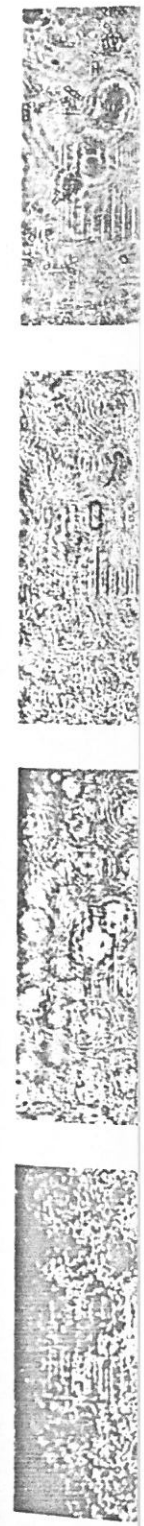
Methods (ii) and (iii) are recommended, as avoiding very fine-grain techniques in the photography. Method (ii) conserves the light well over long distances, and hence exposures are kept low. On the other hand the last method allows further projective magnification to be obtained, at some cost in exposure time, if desired.

We understand that Gabor (Gabor, 1950) has discovered method (ii) independently.

§ 14. Artificial Holograms

Some work has been done on artificial (calculated) holograms, including linear zone-plates, but a discussion of these experiments is reserved for a further paper.

* Note added 5.9.51. This has since been confirmed by reconstruction.



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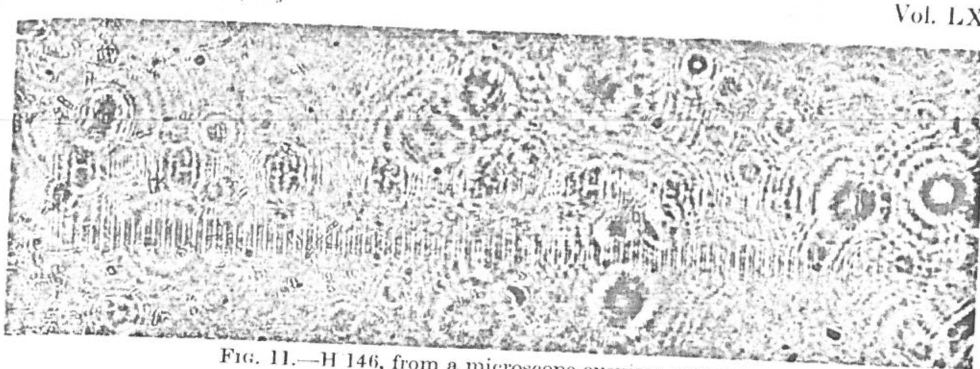

FIG. 11.—H 146, from a microscope eyepiece graticule

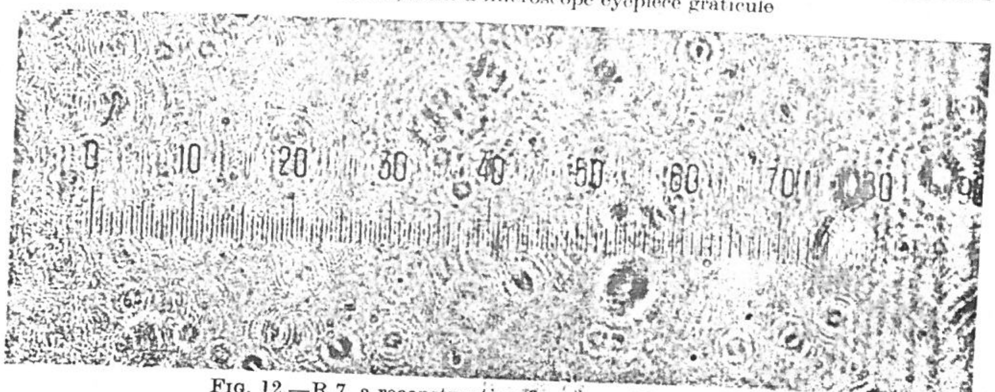

FIG. 12.—R 7, a reconstruction from a positive of H 146

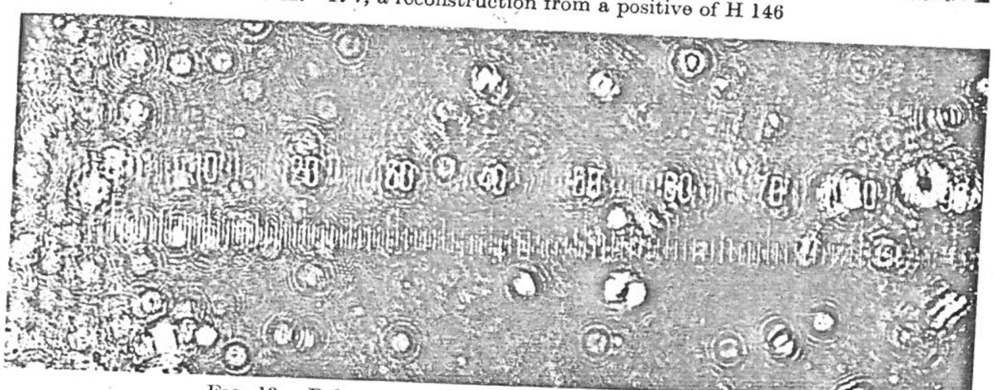

FIG. 13.—R 8, a reconstruction from a negative of H 146

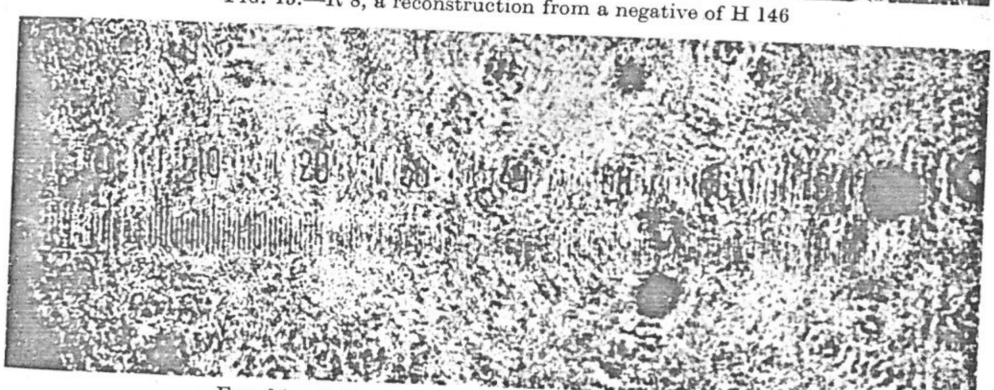

FIG. 14.—R 11, a reconstruction from a positive of R 7 (a third order hologram)

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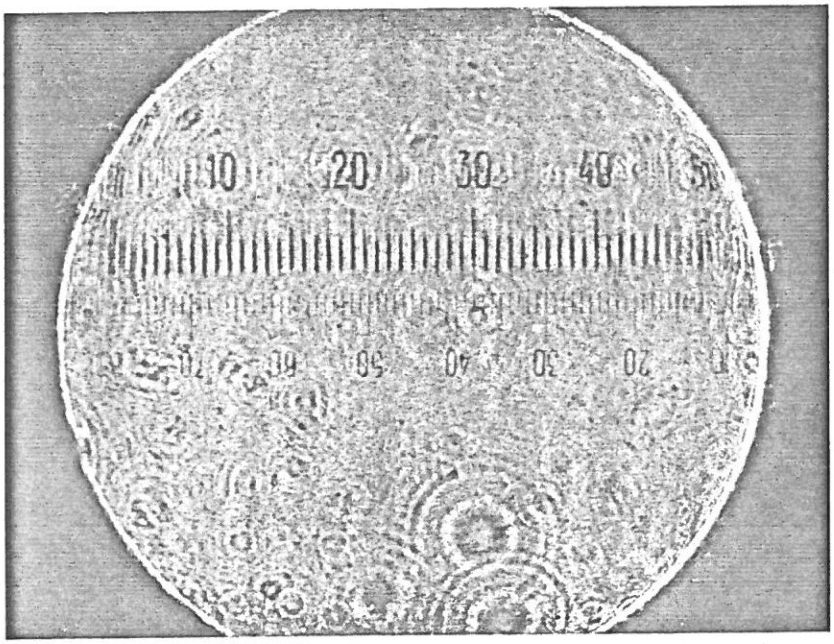

FIG. 10.—Composite image consisting of a reconstruction (above) alongside an image of the object (below), located in the plane of reconstruction, as used for the work of para. 7, using a Gabor system (fig. 9*e*)

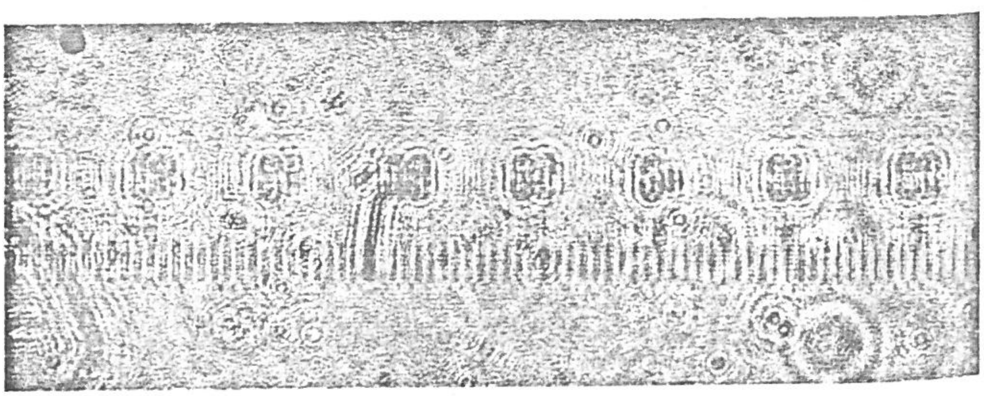

FIG. 15.—H 159, a hologram of a hologram

§ 15. Acknowledgments

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