ON THE REPRODUCTION OF THE OPTICAL PROPERTIES OF AN OBJECT BY THE WAVE FIELD OF ITS SCATTERED RADIATION

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In this paper the formation of a unique kind of optical equivalent of a given object is considered. This optical equivalent is formed by the photographic model, created under certain conditions, of the standing wave which results from superposition of the radiation incident on, and the radiation reflected by, the object. It is shown that if radiation from the same source as is used to make the photograph is incident on such a model (called a "wave photograph"), then the wave function of the radiation reflected by each fixed photographic surface of the standing wave coincides with the wave function of the radiation scattered by the object, in some region of space. The wave function of the radiation reflected by the wave photograph as a whole was found by summation of the wave functions of the radiation reflected by the above-mentioned surfaces. It turned out that this wave function is also related to the wave function of the radiation scattered by the object—it describes the radiation which on being scattered by the object, passed through the wave photograph volume just as if it were filled by a medium with a negative absorption index. The observer, detecting the radiation which is reflected by the wave photograph, sees, accordingly, the spatial image of the object; features of the image, rays of which passed through the photograph volume over a long path, will appear clearer than those features, the rays of which passed through a smaller region of the wave photograph volume. The theory is tested. A special case in which the object was a spherical mirror was studied. In accordance with the theory, the wave photographs of such mirrors were equal in optical strength to the optical strength of the original mirror. In its dispersion properties the photographic layer which was obtained resembled a concave diffraction grating.

Analyzing Gabor's hologram method[1] and Lippman's light photography method,[2] one reaches the conclusion[3] that the phenomena underlying these methods are special cases of the more general phenomenon of the representation of the optical properties of the object by the spatial photographic model of the standing wave which is formed from the superposition of the incident radiation and the radiation scattered by the object. Let us suppose that there exists some arbitrary object O, upon which radiation from source S (Fig. 1a) is incident. For Rayleigh scattering, the radiation reflected by the object, superposed on the radiation issuing from the source, forms a stationary standing wave pattern.* The antinodal surfaces of these waves are designated in Fig. 1 by d_1, d_2, d_3. Into the space surrounding the object, the volume V, filled by a light-sensitive Lippman emulsion, is introduced. After appropriate exposure and chemical processing a photographic image is formed in this volume with a density which is a reproduction of the intensity distribution in the standing wave. It appears that such a spatial structure is a unique kind of optical equivalent of the object. If radiation from the same source which illuminated the object during its exposure is incident upon this structure, then it reflects this radiation in such a way that the wave field of the reflected radiation will be identical, up to some fixed degree, to the wave field of the radiation reflected by the object. Thus, the observer h records the appearance of a virtual spatial image of the object O′ (Fig. 1b). The reproducing properties of the photographic model of the standing wave pattern also extend to the area of spectral composition. If the radiation incident on the object during the making of the photograph is monochromatic, and the radiation incident on the photograph during observation has a complex spectrum, then the photograph reflects only the monochromatic component to which it was exposed.

*Here Rayleigh scattering is that leaving wavelength unchanged.

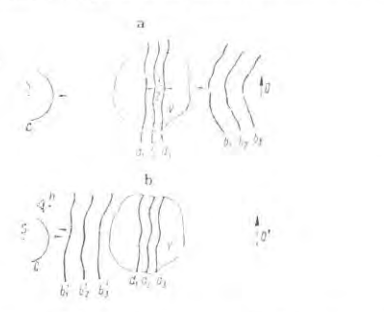

Fig. 1. (a) Schematic diagram showing production of the wave photograph. O—object; c—wave surface of radiation incident on the object; b_1, b_2, b_3—wave surfaces of radiation reflected by object; d_1, d_2, d_3—antinode surfaces of standing wave which is formed as a result of the interference of wave c with waves b_1, b_2, b_3. (b) Schematic diagram showing the formation of image by the wave photograph. d_1, d_2, d_3—The reflecting layers formed at the site of the antinode surfaces; b_1', b_2', b_3'—wave surfaces of radiation reflected by the layers d_1, d_2, d_3; O′—image of the object.

We call this type of photograph a "wave photograph". To produce it, the following conditions must be fulfilled: (a) plane or weakly divergent spherical waves must be incident on the object; (b) the volume of the wave photograph V must be situated at a large distance from the source of radiation and the object; and (c) the density of the photographic deposit must be small and proportional to the intensity of the standing wave.

Let us consider the theory of the process. In accordance with (a) and (b) we write the wave functions of the incident radiation and the radiation scattered by the object.*

*The phase multiplier $e^{i\varphi}$ is hereafter omitted.

$$\psi_s = a_s e^{ikL_s(r)} \tag{1}$$

$$\psi_0 = a_0(r)\, e^{ikL_0(r)}. \tag{2}$$

Here the functions $L_s(r)$ and $L_0(r)$ satisfy the eikonal equation. The wave function of the field which results from superposition of the incident and the scattered radiation has the following form:

$$\psi_W = a_s e^{ikL_s} + a_0(r)\, e^{ikL_0}. \tag{3}$$

Multiplying ψ_W by ψ_W^* we find the intensity of the standing wave

$$I_W = a_s^2 + a_0^2 + 2a_s a_0 \cos k(L_s - L_0). \tag{4}$$

Setting the argument of (4) equal to a constant, kp, we get the equation of the "isophase" surface. For the special case when $kp = 2\pi n$, where n is integral, this surface coincides with the antinode surface.

$$L_s(r) - L_0(r) = p. \tag{5}$$

Substituting (5) into (4) we find

$$I_W = a_s^2 + a_0^2 + 2a_s a_0 \cos kp. \tag{6}$$

We now assume that the total wave field recorded in volume V is such that the density of the photographic deposit q which has been formed is proportional to I_W

$$q = \varkappa I_W. \tag{7}$$

Let us suppose that this deposit is a nonmagnetic dielectric, for which $\mu = 1$ and $\sigma = 0$. For small values of q, the function $\varepsilon(q)$ can be expanded in a series bounded by the first two terms of the expansion

$$\varepsilon_f = \varepsilon_{f0} + \frac{\partial\varepsilon}{\partial q} q = \varepsilon_{f0} + \gamma q, \tag{8}$$

where ε_{f_0} is the dielectric constant of the emulsion prior to exposure. Substituting I_W from (6) into (7) and then substituting the value of q which is obtained, into (8), we find the dielectric constant of the wave photograph

$$\varepsilon_f = \varepsilon_{f0} + \gamma\varkappa\left(a_0^2 + a_s^2 + 2a_s a_0 \cos kp\right). \tag{9}$$

In the volume V we select the infinitesimally thin layer found between the isophase surfaces on which the parameter p has the values p and p + dp (Fig. 2a). Having taken the gradient of (5), it is a simple matter to verify that the normal to this layer is directed along the gradient of ε_f and that, consequently, such a layer may be considered as the boundary separating two media with dielectric constants ε and $\varepsilon + d\varepsilon$. Differentiating (9) with respect to p, we define the quantity $d\varepsilon$

$$d\varepsilon = 2\varkappa\gamma k a_s a_0(\rho) \sin kp\,dp. \tag{10}$$

Here, ρ is the radius vector of the surface layer. Substituting (10) into the Fresnel formula and taking into account the fact that ε_f varies only slightly, we find the "amplitude" reflection coefficient of the layer for natural light, which most closely corresponds to the scalar approximation we assumed,

$$d\mu = \zeta a_0(\rho) \sin kp\,dp, \tag{11}$$

where

$$\zeta = \frac{1}{2}\frac{\varkappa\gamma}{\varepsilon} k a_s.$$

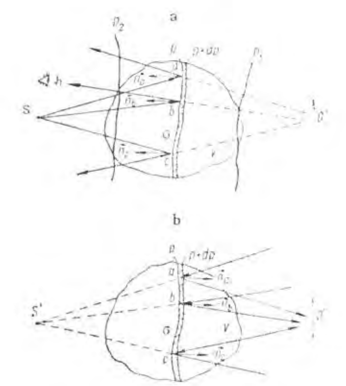

Fig. 2. (a) Radiation from the same source S as illuminates the object during the making of the wave photograph is incident on the isophase surface. Sa, Sb, Sc—the rays incident on the isophase surface; n_a, n_b, and n_c—normals to the isophase surface, O′—virtual image of the object. (b) Radiation which forms a real image of the source S is incident on an isophase surface of the wave photograph. S′—the real image of the source, and O′—real image of the object.

Let us suppose now that the same radiation ψ_s which was incident on the object during the making of the photograph is incident on the wave photograph (Fig. 2a). Taking advantage of the low density of the photographic image we determine the wave function of the radiation reflected from each individual isophase layer, in order to later combine these functions to determine the function of the incident radiation reflected by the entire photograph.*

*An analogous calculation technique is described in ref. 2.

Multiplying the wave function of the incident radiation ψ_s by the reflection coefficient of the layer $d\mu$ we find the value which the wave function $d\psi_f$ of the radiation reflected by this layer assumes at the surface of the layer itself. In other words we find the boundary condition of the Dirichlet problem for the wave function $d\psi_f$

$$d\psi_f(\rho) = d\mu\psi_s. \tag{12}$$

Substituting into (12) the value of $d\mu$ from (11) and ψ_s from (1), and also using the equation of the isophase surface (5), we get

$$d\psi_f(\rho) = \zeta a_s a_0 e^{ikL_0(\rho)} e^{ikp} \sin kp\,dp. \tag{13}$$

It is easy to see that $d\psi_f(\rho)$ differs from the wave function, ψ_0 of (2), of the radiation reflected from the object, only by the constant multiplier

$$\chi = \zeta a_s e^{ikp} \sin kp\,dp \tag{14}$$

Should the surface of the isophase layer be closed, then from this coincidence, on the basis of the theorem of the uniqueness of the relation of the wave function to the boundary conditions, it would have been possible to conclude that the wave function of the radiation reflected by the isophase layer coincides with the wave function ψ_0 of the radiation scattered by the object with accuracy up to the multiplier χ. Solution of the problem for a nonclosed surface is substantially simplified when the dimensions of the isophase layer are much greater than the wavelength. In this case the well-known hypothesis of Kirchhoff diffraction theory– involving the fact that in calculating the perturbation behind a screen with apertures, one may take the boundary conditions on the aperture surface as the boundary conditions corresponding to free propagation of light, and the boundary conditions on the surface adjacent to the screen as equal to zero–becomes completely accurate.[4] Taking the inverse of this assumption we conclude that the wave function of the radiation reflected by the isophase layer $d\psi_s(r)$ which satisfies the boundary condition (13) coincides with the wave function of the radiation which, being reflected from the object, passed through the surface of the isophase layer as if it were passing through an opening in an opaque screen. We denote such a function as $d\psi_0'(r)$ and we write

$$d\psi_f(r) = d\psi_0'(r) \tag{15}$$

The wave function $d\psi_0'$ coincides with the wave function ψ_0 only in the sector of space bounded by the cone connecting the object and the edge of the isophase layer, i.e., in this case

$$d\psi_0'(r) = \chi\psi_0(r). \tag{16}$$

In the remaining portion of space, this function, with accuracy up to the diffraction limit is equal to zero, i.e.,

$$d\psi_0'(r) = 0. \tag{17}$$

In accordance with what has been said above, the observer h (Fig. 2a), registering the radiation reflected by the isophase layer, sees the virtual space image of the object O′, situated at the same place which this object occupied when the photograph was made. Thereby the surface layer will seem to him a window set in a space of objects.

We consider the mechanisms of reflection of the radiation from the isophase layer from the point of view of geometrical optics. Having taken the gradient of the left side of (5) and taking into account that $\nabla L = 1$ one may obtain the following relation:

$$\mathbf{n} = \frac{\mathbf{l}_s - \mathbf{l}_0}{|\mathbf{l}_s - \mathbf{l}_0|}, \tag{18}$$

where n is the normal to the isophase layer, $\mathbf{l}_s$ is the ray vector of the incident radiation and $\mathbf{l}_0$ is the ray vector of the radiation scattered by the object. On the other hand, the law of reflection of radiation from the interface between two media may be written in this form:

$$\mathbf{n} = \frac{\mathbf{l}_s - \mathbf{l}_f}{|\mathbf{l}_s - \mathbf{l}_f|}, \tag{19}$$

where $\mathbf{l}_f$ is the ray vector of the radiation reflected by the boundary. Equating (18) to (19), we get $\mathbf{l}_0 = \mathbf{l}_f$, i.e., the direction of the rays reflected by the isophase layer coincides with the direction of the rays reflected by the object. Since the amplitude of the rays reflected by the isophase layer also coincides with the amplitude of the rays reflected by the object (11), then the observer sees a virtual image of the object O′. The corresponding path of the rays is shown in Fig. 2a.

A wave photograph may form not only a virtual image of the object, but a real one as well. Such an image is obtained when the photograph during observation is illuminated by radiation which converges into the real image of the source. To show this, it is necessary to vary the sign of the eikonal of the wave function of the incident radiation and to carry out the transformation analogous to (12) and (13). The corresponding case is shown in Fig. 2b.

We proceed to consideration of the interaction of the radiation with the wave photograph in its entirety. To do this, we generalize the theory to the case when the wave photograph, during observation, is illuminated by a source with a complex spectrum, i.e., we assume that

$$\psi_s = \int_0^\infty a_s(k')\, e^{ik'L_s(r)}\, dk'. \tag{20}$$

Substituting for ψ_s in (12) and using Eqs. (5) and (11), we find the value which belongs in the given case to the wave function of the radiation reflected by the isophase layer at the surface of the layer itself

$$d\psi_f(\rho) = \zeta a_0(\rho) \sin kp\, dp \int_0^\infty a_s(k')\, e^{ik'L_0(\rho)} e^{ik'p}\, dk'. \tag{21}$$

It is not difficult to note that (21) is the sum of functions the eikonal of which is the same as the eikonal of the radiation scattered by the object. Accordingly we may find the value assumed at the point h by the wave function of the radiation reflected by the isophase layer by replacing the radius vector of the points of the isophase surface ρ by the radius vector of the observation point r of (13), (15), (16) just as in the previous case. The wave function of the radiation reflected by the entire wave photograph is found by summing the wave functions of the radiation reflected by the separate isophase layers. In this, one need take into account only the layers for which the cone connecting the edge of the layer and the object includes the point of observation; see (16) and (17). In the case shown in Fig. 2a, such an isophase layer is included between the layers p_1 and p_2, passing through the points in which the straight line connecting the object and the point of observation cuts the surface of the volume of the wave photograph. Substituting r for ρ in (21) integrating this expression from p_1 to p_2, signifying by ψ_g the integral with respect to the boundary of the volume where ε_f undergoes a discontinuity and d does not satisfy (11), and representing sinkp in exponential form, we obtain the wave function of the radiation reflected by the entire wave photograph,

$$\psi_f(r) = \psi_g + \zeta a_0(r) \int_{p_1}^{p_2} dp\, \frac{e^{ikp} - e^{-ikp}}{2i} \int_0^\infty a_s(k')\, e^{ik'L_0(r)} e^{ik'p}\, dk'. \tag{22}$$

The first term of this expression is the wave function of the radiation reflected by the boundary of the photograph volume. We consider the second term more carefully, designating it ψ_0''. Changing the order of the integration with respect to p and k′ we get

$$\psi_0'' = -\frac{i}{2}\zeta a_0(r) \int_0^\infty dk' a_s(k')\, e^{ik'L_0(r)} \int_{p_1}^{p_2} \left[e^{i(k+k')p} - e^{i(k'-k)p}\right] dp.$$

Carrying out the integration with respect to p we find

$$\psi_0'' = -\frac{i}{2}\zeta a_0(r)\int_0^{\infty} a_s(k')\,e^{ik'L_0(r)}\left[\delta_1(k+k') - \delta_1(k'-k)\right]dk', \quad (23)$$

where

$$\delta_1(k'-k) = 2e^{\frac{i(p_2+p_1)}{2}(k'-k)}\,\frac{\sin\frac{(p_2-p_1)(k'-k)}{2}}{(k'-k)}.$$

The functions $\delta_1(k + k')$ and $\delta_1(k' - k)$ have the properties of Dirac delta functions. The function $\delta_1(k' - k)$ has a maximum for $k' = k$, where it assumes the value $p_2 - p_1$. Analogously, $\delta_1(k + k')$ has a maximum for $k' = -k$. The width of these maxima is determined by the dependence

$$\Delta k' = \frac{2\pi}{p_2 - p_1}. \quad (24)$$

Inasmuch as in our case the dimensions of the volume of the wave photograph are much greater than a wavelength, i.e., $p_2 - p_1 \gg 1/k$ when $\Delta k' \ll k$ and the delta functions may be replaced by rectangular pulses of width Δk and height $p_2 - p_1$. Substituting this function into (23) and transforming with the use of the derived expression (22), we find

$$\psi_f(r) = \psi_0 + \frac{i}{2}\zeta(p_2 - p_1)\,a_0(r)\int_{k'=k-\frac{\Delta k}{2}}^{k'=k+\frac{\Delta k}{2}} a_s(k')\,e^{ik'L(r)}dk'. \quad (25)$$

Thus, the wave function of the radiation reflected from the wave photograph is made up from the wave function of the radiation reflected by the boundary of its volume, and the packet of wave functions of which the wavelength differs little from the wavelength of the radiation which exposed the photograph; its eikonal agrees with the eikonal of the radiation scattered by the object; its amplitude is proportional to the amplitude of the radiation scattered by the object, and the path $(p_2 - p_1)$, which this radiation traversed in the volume of the wave photograph. All these signs give the right to identify such radiation as that which, being scattered by the object, passed through the volume of the wave photograph, as through a medium with a slightly negative absorption. The observer detecting such radiation sees a virtual spatial light image of the object; moreover, the features of the object the rays from which traveled through the volume over a long distance will seem clearer than the features the rays from which intersect the volume where its spatial extension is small.

Changing the limit of integration in (20) with respect to k′ to $-\infty$ and making all the subsequent transformations, one may show that the wave photograph also forms a real image of the object.

It must be noted that the more complete theory gives for ψ_f an expression differing from (25) by the presence of a term which corresponds to the radiation being propagated in the region of the geometric shadow of the object. The reason for the absence of this term in the present case is sufficiently clear: in the region of the geometric shadow of the object, the isophase surfaces degenerate into lines, and the mathematical apparatus based on them no longer has any meaning.

To confirm the theory, a series of experiments was set up. Below we present the results for a special case when a spherical mirror was used as an object. Being the optical equivalent of the object, the wave photographs of this mirror must according to the theory, reproduce the optical strength and resolving power of the original for the same wavelength as that for which the exposure was carried out.

The scheme of the wave photograph obtained with the spherical mirror is shown in Fig. 3a. Above the convex aluminized mirror Z a photo-plate a is situated. The normal to the surface of the photoplate makes an angle α with the optic axis of the collimator, which is located above the system. The plane monochromatic wave which emerges from the collimator, on passing through the photo-plate is reflected from the mirror. As a result of the superposition of the incident and reflected radiation, above the mirror a system of standing waves is formed, which is recorded by the emulsion layer e.

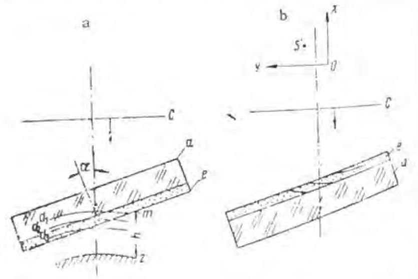

Fig. 3. (a) Diagram of the production of wave photograph of convex mirror. c–Wave surface of incident radiation, a–photo-plate, e–emulsion layer of photo-plate, d_1, d_2, d_3–antinode surfaces of the standing waves, Z–convex mirror with outer aluminizing. (b) Diagram of measurement of optical characteristics of wave photograph of convex mirror. S′–image of monochromator slit or hatched globe, formed by wave photograph. XOY is the coordinate system in which the position of the image is measured.

The photography is carried out on a special Lippmann photo-plate which had been hypersensitized.[5] Chemical processing of the exposed photo-plate is achieved by two methods. In the first of these the exposed plate, after development in a metol-hydroquinone developer and and fixing, was bleached with mercuric chloride. As a result, in the volume of the photograph, clear dielectric in an amount approximately proportional to the intensity of the standing wave was formed. See (7), (8). In development by the second method the exposed plate is developed in a pyrogallic developer. As a result metallic silver with a high coefficient of reflection was formed at the standing wave antinodes.

The fact that the optical properties of the photographs thus obtained duplicate the optical properties of their originals, could easily be seen even during a simple visual observation. Thus, if the radiation of an incandescent lamp is incident on the photograph from the side of the emulsion layer, then after reflection it is collected in a real image of the filament of this lamp; if the same radiation were to fall from the other side, then the image of the filament would be virtual (Figs. 2a and 2b). The chromaticity of the images of the filament coincided with that of the radiation which exposed the photo-plate.

In Fig. 4 we present curves showing the dependence of the reflection coefficients of certain photographs on the wavelength. The setup for these measurements was that of Fig. 3b. Incident on the photograph from the side of the emulsion layer was radiation emanating from a monochromator slit which was situated in the focal plane of the collimator objective. The radiation reflected by the photograph converged in a real image of the slit S'. This image was projected on a photomultiplier which registered the value of the current of the reflected radiation. After this, in the position of the wave photograph an arc shaped mirror was set, equal in dimension and radius of curvature to the convex original mirror, and the current of the radiation scattered by this mirror was found. As may be seen from Fig. 4, the reflection coefficient of the wave photograph is small but has a clearly expressed maximum in the region approximately corresponding to the wavelength of the radiation which exposed the photograph. A certain shifting of the maximum in the region of short wavelengths is explained by the shrinkage of the layer, which occurs during the chemical processing.

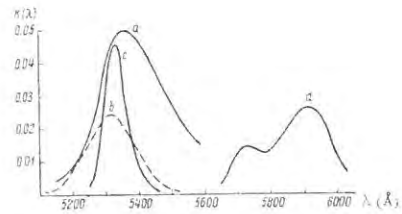

Fig. 4. Dependence of the reflection coefficient of wave photographs of convex mirrors on the wavelength.

Curve	Wavelength of the radiation which exposed the photograph (Å)	Thickness of emulsion layer (μ)	Type of chemical processing
a	5460	4	pyrogallol
b	5460	4	bleaching
c	5460	10	bleaching
d	5890	4	bleaching

We measured, in addition, the resolving power of the wave photographs obtained for $\alpha = 0^\circ$. The general setup of this measurement was similar to the one shown in Fig. 3b with the sole difference that in the focal plane of the collimator objective in the given case a hatched globe was placed, which was illuminated by radiation from a mercury lamp (λ = 5460 Å). Investigation of the image of the globe S' showed that the resolving power of the wave photographs differed little from the resolving power of their originals. Thus, if at field of view angles from 0° to 5.4° the resolving power of the mirror of radius 2058 mm both for sagittal and for meridional rays was 53 lines/mm, then the resolving power of the wave photograph in the same conditions was 48 lines/mm. Similarly, for a radius of curvature of the original of 1000 mm and resolving power of 110 lines/mm, the resolving power of the wave photograph was 80 lines/mm.

The study of the wave photographs of convex mirrors was completed by measuring the dependence of their focal lengths on the wavelength. The device setup in this case was similar to the one for the measuring of the spectral curves of the reflection coefficient (Fig. 3b). We judged the focal distances of the wave photograph according to the position of the image of the monochromator slit S' in the coordinate system XOY. Here, as the origin of the coordinate system, the point was chosen which the image of the slit would pass through when in place of the wave photograph, a curved mirror with a radius of curvature equal to the radius of curvature of the mirror original is put.

In Fig. 5 is shown the result of the measurement of the focal length of the wave photographs of mirrors of different radii of curvature. The photographs were made for $\alpha = 0$ and λ = 5460 Å. Wavelength is given along the ordinate axis of the figure, and the point λ = 5460 Å was taken as the ordinate origin. Along the abscissa the displacement of the images of the slits along the axis OX of the chosen coordinate system (Fig. 3b) is given; the image displacement along the axis OY is absent in this case.

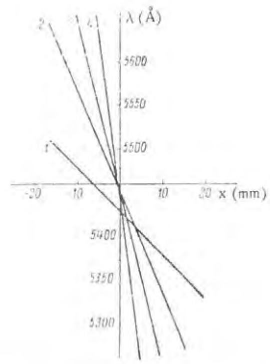

Fig. 5. Dependence of the focal lengths of the wave photographs on the wavelength for different radii of curvature of the original mirror. The surface of the photoplate was normal to the incident radiation, and λ = 5460 Å. 1—R = 2058 mm, 2—R = 1000 mm, 3—R = 600 mm, 4—R = 300 mm.

Figure 6 shows the results of measurements of the position of the image of the slit when it was inclined with respect to the incident radiation ($\alpha \neq 0$, Fig. 3a) during exposure of the photoplate. The coordinate axes of the graph correspond to the axes represented in Fig. 3b, and the experimental curves are the trajectories which describe in this system of coordinates the image of the slit upon variation of the wavelength of the radiation which forms it. On the curves the point at which the image of the slit is found for λ = 5460 Å is sharply separated.

It is easy to see, that thanks to the special choice of the coordinate system XOY, in which the position of the slit is taken into account, and of the coordinate systems of the graphs, the experimental curves shown in Fig. 5 pass through the origin of coordinates only when the wave photographs corresponding to them accurately reproduce the focal length of the original mirror for the monochromatic radiation with wavelength equal to the wavelength of the radiation incident on the photograph during the making of the photograph. In Fig. 6, the coincidence of the point λ = 5460 Å with the origin of coordinates corresponds to such a case. As is seen from these graphs, the wave photographs we obtained reproduce the focal length of the original to an accuracy of not less than 0.6%.

We treat some questions related to the practical application of the photographs we made. As is seen from Fig. 5, the wave photographs exposed to the radiation incident normal to the surface of the photoplate have an extraordinarily strong longitudinal chromatic

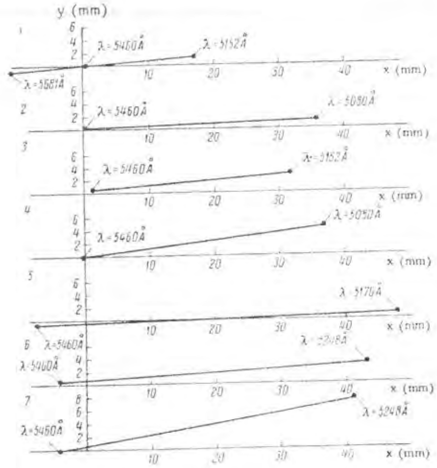

Fig. 6. Spectral disposition of the wave photographs of the convex mirrors obtained for different angles of inclination of the photoplate during exposure; λ = 5460 Å. 1—R = 600 mm; α = 1°47'; 2—R = 1000 mm, α = 58'; 3—R = 1000 mm, α = 2°35'; 4—R = 1000 mm, α = 3°35'; 5—R = 2058 mm, α = 58'; 6—R = 2058 mm, α = 1°47'; 7—R = 2058 mm, α = 5°11'.

aberration with sign opposite to the sign of the aberration of an ordinary lens. The fact that elements with such properties may find application in optical device design should not be overlooked.

For the wave photographs exposed with inclined positions of the photoplate, there is added to the longitudinal chromatic aberration a transverse aberration which increases with angle of inclination of the plate. It is not difficult to show that the magnitude of this aberration is determined by the same laws as hold for diffraction gratings. Actually, using the known relations for the first-order spectra of the usual diffraction grating, one may find t_1, the grating constant for the grating, the dispersion of which is equal to the dispersion of the given photograph. On the other hand, knowing the inclination of the photoplate and the distance between the antinodes of the standing wave, one may determine the period t_2 of the surface structure of the wave photograph directly. The table gives the values of t_1 and t_2 calculated by this means for each of the photographs represented in Fig. 6. The fact that these values come out in good agreement shows that the mechanism of the interaction of the radiation with the wave photograph is analogous to the mechanism of interaction of the radiation with a diffraction grating. Of course, there are differences too. They find their expression in the fact, for example, that the spectrum in the given case does not lie on the Rowland circle.

Radius of curvature of the original mirror (mm)	Angle α	Photographic constant t_1, calculated with respect to the dispersion (μ)	Number of striations per millimeter corresponding to t_1	Photographic constant t_2 calculated from the standing wave geometry (μ)	Number of striations per millimeter, corresponding to t_2
600	1°47'	7.1	140	8.7	115
1000	58'	17.2	58	16.1	62
1000	2°36'	6.4	156	6.0	166
1000	3°35'	4.1	244	4.3	230
2058	58'	21.0	48	16.1	62
2058	1°47'	8.1	123	8.7	115
2058	5°11'	2.8	356	3.0	330

It should be noted that the parameters of the diffraction gratings we obtained are not limited. By increasing the angle of inclination of the plates, one could have prepared grating lattices with still greater dispersion. The fundamental difficulty which is encountered in this method consists of the absence of sufficiently bright sources of monochromatic radiation. Actually in producing the given photographs the maximum permissible dimensions of the source and the degree of its monochromaticity were determined according to the formulas

$$\Delta S = \frac{\lambda R f}{2hd} \tag{26}$$

and

$$\Delta\lambda = \frac{\lambda^2}{4h}. \tag{27}$$

Here ΔS is the dimension of the radiation source, f is the focal distance of the collimator, h is the depth of the standing wave pattern, R is the radius of curvature of the original mirror, d is the diameter of the original mirror, and $\Delta\lambda$ is the permissible width of the spectrum of the source radiation. From these relations it follows that the increase in the depth of the standing wave pattern (Fig. 3a) connected with the increase in the angle of inclination, is attained on account of the diminution of the dimensions of the source and the rise in its monochromaticity. However, the intensity of the standing wave pattern is decreased by this, and the exposure time grows beyond reasonable limits. Substantial progress in this direction should result from the use of quantum generators, which provide high-intensity radiation and are highly monochromatic. The practical application of wave photographs as diffraction gratings may also be precluded by the smallness of their reflection coefficients. It is easy to see, however, that if in the surface layer of the wave photograph the unexposed emulsion is removed, then a relief is formed, which is typical of a grating with a concentration of intensity in the given part of the spectrum. Metalizing such a relief plate, one may obtain a diffraction grating with a high reflection coefficient.

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