Ben FT.

on the MATHEMATICAL OPTICS of WHITE-LIGHT TRANSMISSION HOLOGRAMS

(Rainbows, Multi-Colored, and Achromats)

by

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and the
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with acknowledgments to:
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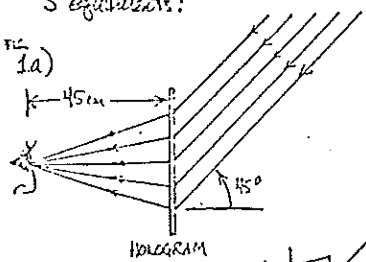
Richard Silberman
Robert Wechsler (dec.)

GOAL: To make a rainbow hologram that gives a GREEN image straight ahead at arm's length when viewed with the Sun at 45° above

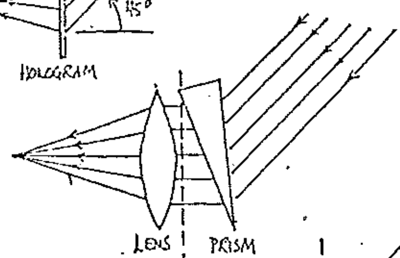
I. HOLOGRAPHIC OPTICS

3 equivalents:

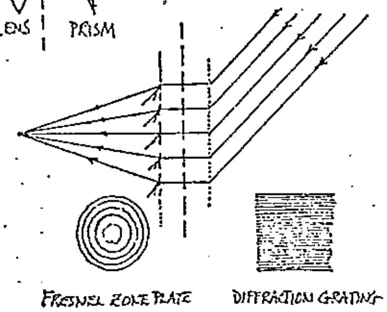
FIG. 1a)



1b)

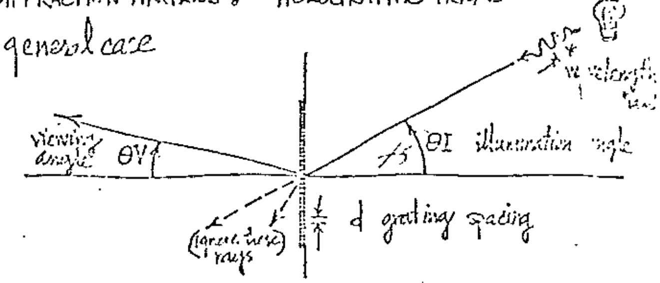


1c)



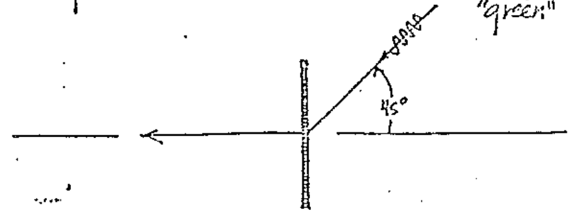
DIFFRACTION ANGLES: HOLOGRAPHIC PRISMS

2) general case



$$\sin\theta V + \sin\theta I = \lambda/d$$

3) example $\theta V = 0$, $\theta I = 45°$, $\lambda = 0.55\,\mu m$ "green" $d = ??$



$$\sin 0 + \sin 45 = .55/d$$

$$d = \frac{.55}{0 + .707} = 0.78\,\mu m$$

"spatial frequency" $f = \frac{1000}{d} = 1300\ \frac{cy}{mm}$

what about the "red" and "blue"?

③

Where is the RED ($\lambda = 0.63\ \mu m$)?

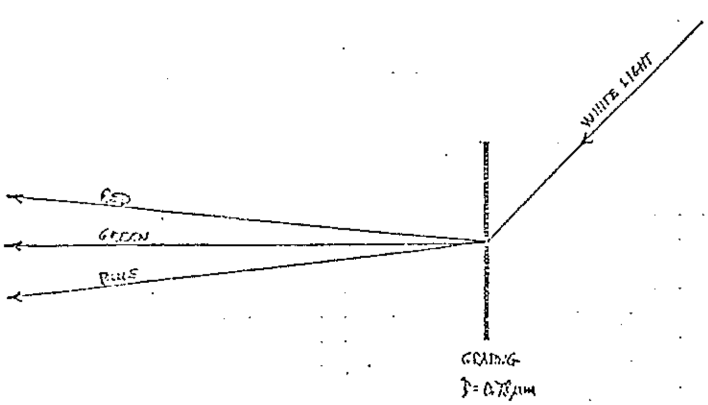
$$\sin\theta_{RED} + \sin 45 = .63/.78$$

$$\theta_{RED} = \sin^{-1}(.81 - .71) = 5.8°$$

where is the BLUE ($\lambda = 0.47\ \mu m$)?

$$\sin\theta_{BLUE} + \sin 45 = .47/.78$$

$$\theta_{RED} = \sin^{-1}(.60 - .71) = -6.0°$$

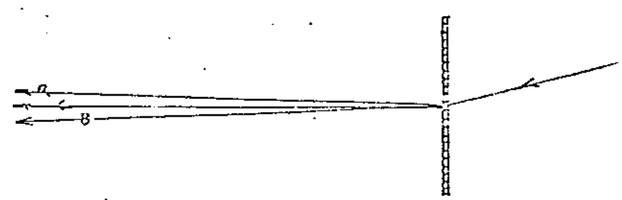


nb. 2-digit results presented not internally consistent

other examples

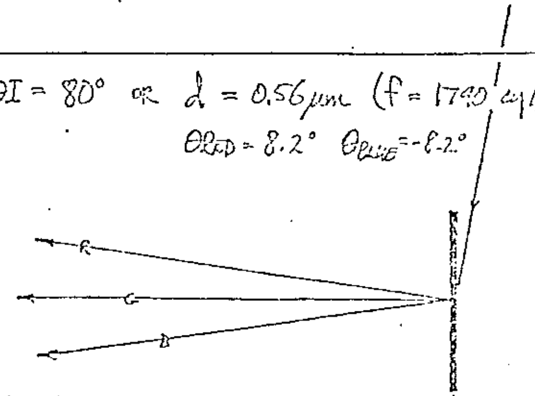
$\theta I = 15°$ OR $d = 2.1\ \mu m$ ($f = 470$ cy/mm)

$\theta_{RED} = 2.2°$ $\theta_{BLUE} = -2.2°$



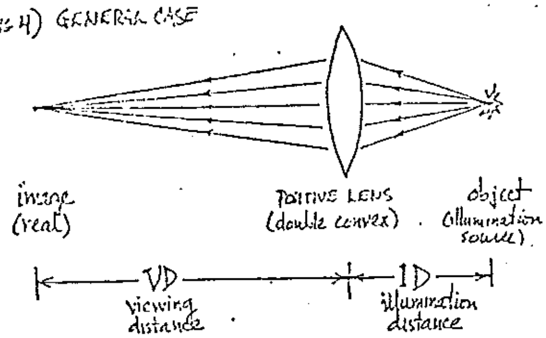
$\theta I = 80°$ OR $d = 0.56\,\mu m$ ($f = 1790$ cy/mm)

$\theta_{RED} = 8.2°$ $\theta_{BLUE} = -8.2°$



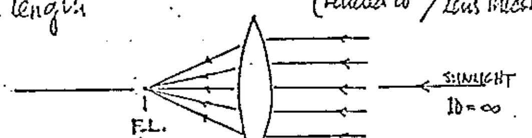
REFRACTION FOCUSING: GLASS LENSES

FIG 4) GENERAL CASE

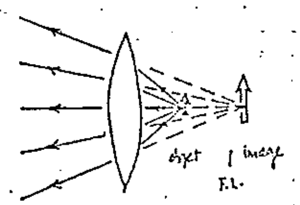


focusing rule $\frac{1}{VD} + \frac{1}{ID} = \frac{1}{F.L.}$

F.L. = focal length (related to 1/lens thickness)



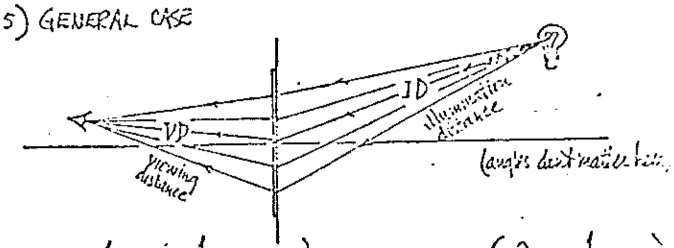
virtual image
(VD = −5)



DIFFRACTION FOCUSING: HOLOGRAPHIC LENSES

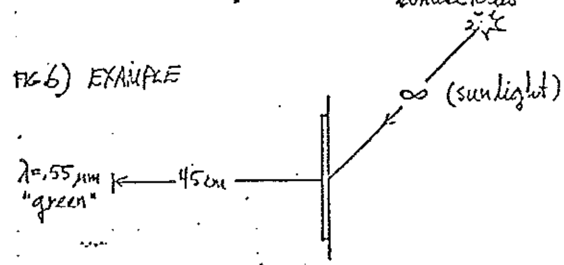
(usually "astigmatic" – limited here to horizontal focusing)

FIG 5) GENERAL CASE



$$\frac{1}{VD} + \frac{1}{ID} = \frac{\lambda}{LF}$$ "lambda focus" $\left(= \frac{\lambda_2}{\lambda_1} \frac{1}{\text{Focal Length}}\right)$

FIG 6) EXAMPLE



$$\frac{1}{45} + \frac{1}{\infty} = \frac{\lambda}{LF}$$

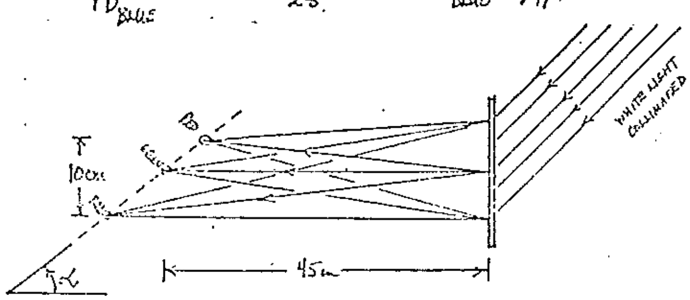
$$LF = \left(\frac{.55}{1/45 + 0}\right) = 25 \quad \mu m \times cm$$

where is the RED ($\lambda = .63\,\mu m$)?

$$\frac{1}{VD_{RED}} + 0 = \frac{.63}{25} \;;\; VD_{RED} = \frac{25}{.63} = 40\text{ cm}$$

where is the BLUE ($\lambda = .47\,\mu m$)?

$$\frac{1}{VD_{BLUE}} + 0 = \frac{.47}{25} \;;\; VD_{BLUE} = \frac{25}{.47} = 53\text{ cm}$$



spectrum tip angle, $\alpha = 35°$

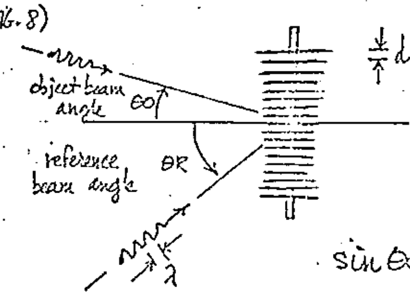
from another analysis $\tan\alpha = \sin\theta I$

table 1:

θI	30°	45°	60°
α	26½°	35°	41°

INTERFERENCE: MAKING A HOLO-PRISM

FIG. 8)



$$\sin\theta O + \sin\theta R = \frac{\lambda}{d}$$

to get a particular d (= 0.78 μm) in RED light:
(compensate for GREEN→RED wavelength change)

OPTION A: fix OBJECT beam $\theta V = \theta O (= 0°)$

$$\sin 0 + \sin \theta R = .63/.78$$

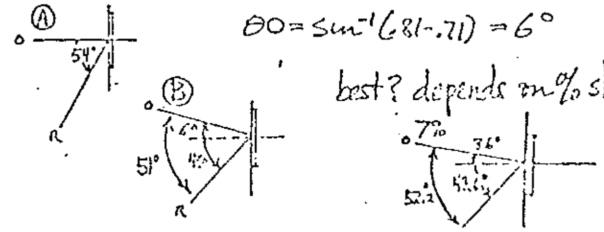
$$\theta R = \sin^{-1}(0.81) = 54°$$

OPTION B: fix REFERENCE beam $\theta R = \theta I (= 45°)$

$$\sin\theta O + \sin 45 = .63/.78$$

$$\theta O = \sin^{-1}(.81 - .71) = 6°$$

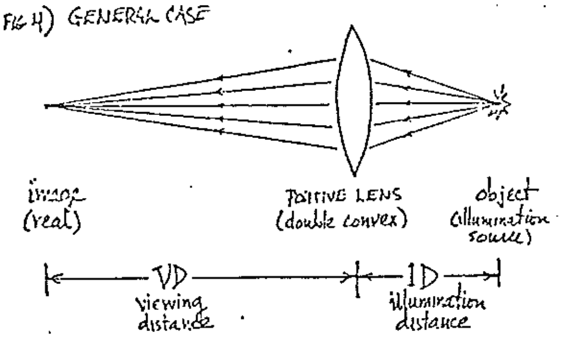
best? depends on % shrinkage



⑤

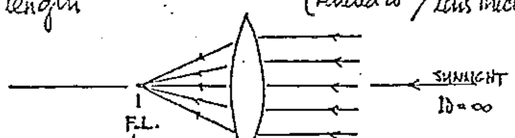
REFRACTION FOCUSING: GLASS LENSES

FIG 4) GENERAL CASE

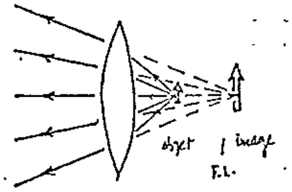


focusing rule $\frac{1}{VD}+\frac{1}{ID}=\frac{1}{F.L.}$

F.L. = focal length (related to 1/lens thickness)



virtual image (VD = −5)

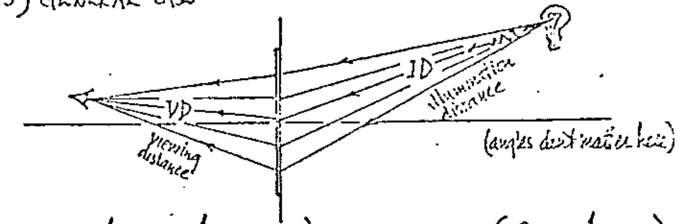


⑥

DIFFRACTION FOCUSING: HOLOGRAPHIC LENSES

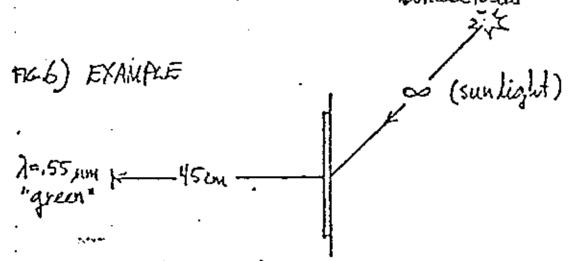
(usually "astigmatic" - limited here to horizontal focusing)

FIG.5) GENERAL CASE



$\frac{1}{VD}+\frac{1}{ID}=\frac{\lambda}{LF}$ "lambda focus" $\left(=\frac{\lambda_2}{\lambda_1}\,\frac{1}{\text{Focal Length}}\right)$

FIG.6) EXAMPLE



$\frac{1}{45}+\frac{1}{\infty}=\frac{1}{LF}$

$LF=\left(\frac{.55}{1/45+0}\right)=25$ μm × cm

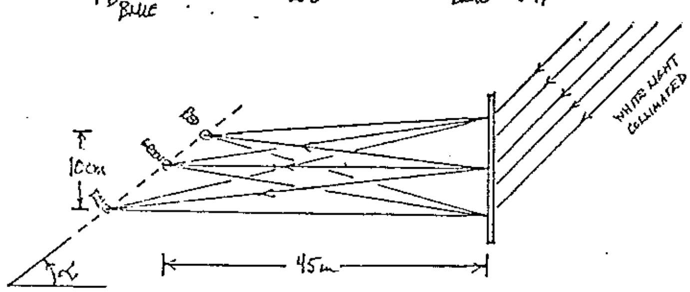
⑦

where is the RED $(\lambda=.63\,\mu m)$?

$\frac{1}{VD_{RED}}+0=\frac{.63}{25}$; $VD_{RED}=\frac{25}{.63}=40$ cm

where is the BLUE $(\lambda=.47\,\mu m)$?

$\frac{1}{VD_{BLUE}}+0=\frac{.47}{25}$; $VD_{BLUE}=\frac{25}{.47}=53$ cm



spectrum tip angle, $\alpha = 35°$

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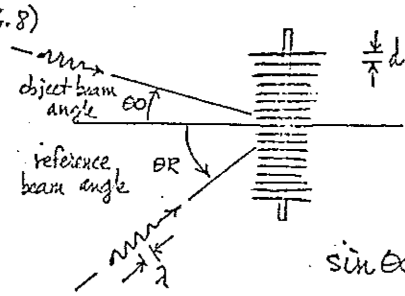
⑧

INTERFERENCE: MAKING A HOLO-PRISM

FIG. 8)



$\sin\theta O+\sin\theta R=\frac{\lambda}{d}$

to get a particular d (=0.78 μm) in RED light? (compensate for GREEN→RED wavelength change)

OPTION A: fix OBJECT beam $\theta V=\theta O (=0°)$

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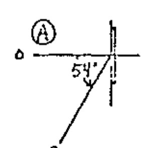
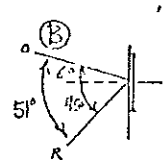
$\theta R=\sin^{-1}(0.81)=54°$

OPTION B: fix REFERENCE beam $\theta R=\theta I\ (=45°)$

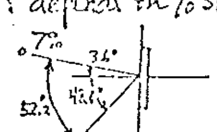
$\sin\theta O+\sin 45 = .63/.78$

$\theta O=\sin^{-1}(.81-.71)=6°$



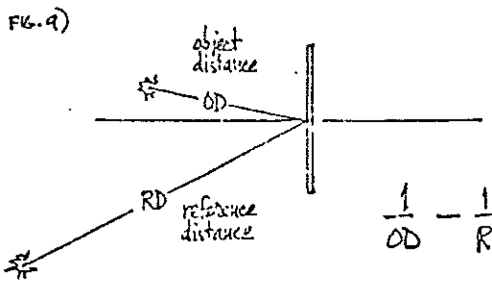


best? depends on % shrinkage



INTERFERENCE : MAKING A HOLO-LENS

FIG. 9)



$$\frac{1}{OD} - \frac{1}{RD} = \frac{\lambda}{LF}$$

* note the minus sign here

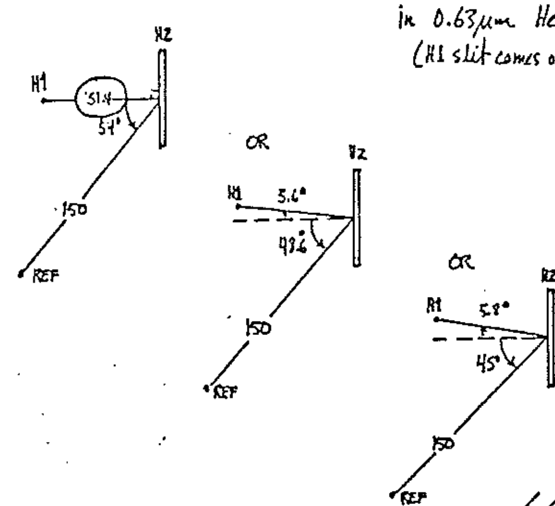
to get a particular LF (= 25)
use maximum (table limited) RD = 150 cm (5')

$$\frac{1}{OD} - \frac{1}{150} = \frac{.63}{25}$$

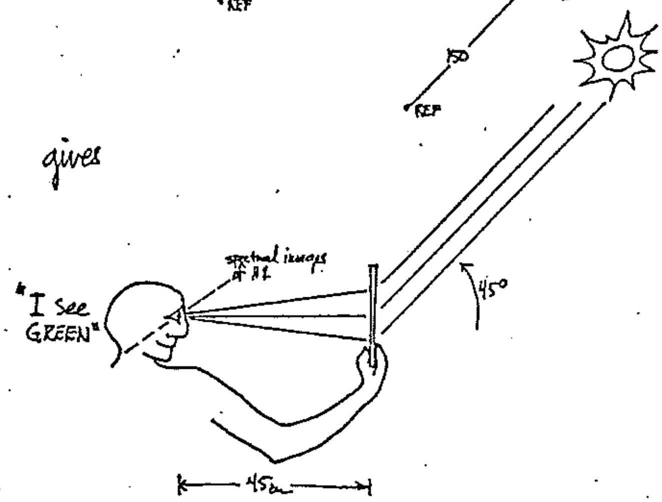
$$OD = \frac{1}{\frac{.63}{25} + \frac{1}{150}} = 31.4 \text{ cm}$$

II. RAINBOW HOLOGRAMS

in 0.63 μm He-Ne light
(H1 slit comes out of the page)

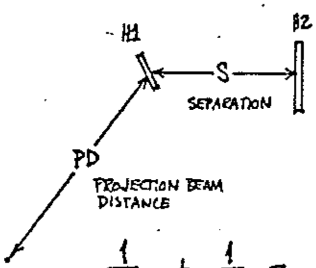


gives



NEXT STEP:
How to produce an H1 that projects at 31.4 cm?
WITHOUT COLLIMATORS (RD = PD = as big as possible)

projection in general
FIG. 11)



H1 as a lens - focusing law

$$\frac{1}{PD} + \frac{1}{S} = \frac{\lambda}{LF_{H1}}$$

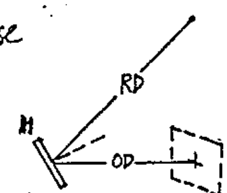
in particular:
if S must be 31 cm, and PD must be 150 cm

$$\frac{1}{150} + \frac{1}{31} = \frac{.63}{LF_{H1}}$$

$$LF_{H1} = \frac{.63}{\frac{1}{31} + \frac{1}{150}} = 16.2 \ \mu m\, cm$$

exposure geometry for H1 to get an LF_{H1}

general case
FIG. 12)



(tip master plate to bisect beam angle)

holo-lens again

$$\frac{1}{OD} - \frac{1}{RD} = \frac{\lambda}{LF}$$

so: if LF must be 16.2 and RD = 150 cm

$$\frac{1}{OD} - \frac{1}{150} = \frac{.63}{16.2}$$

$$OD = \frac{1}{\frac{.63}{16.2} + \frac{1}{150}} = \underline{\underline{22 \text{ cm}}}$$

farther than for a full-aperture image plane transfer, nearer than one's lens

the direct route:

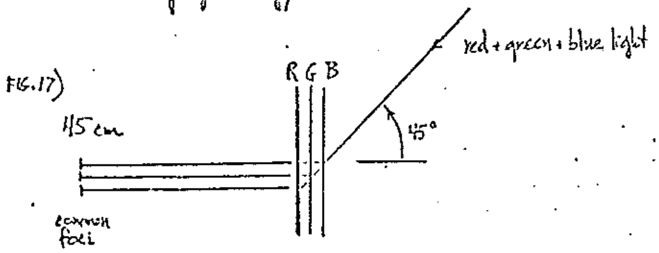
$$\frac{1}{OD} = \frac{1}{S} + \frac{1}{PD} + \frac{1}{RD} = \frac{1}{31} + \frac{1}{150} + \frac{1}{150} = \frac{1}{22}$$

(11)

III. MULTI-COLOR RAINBOWS (pseudo-color + simple achromats)

3 co-existing gratings/lenses (all in the same emulsion)

FIG. 17)



	GRATING SPACING	LENS "λ-FOCUS"
RED:	$d_{RED} = \frac{.63}{\sin 45} = 0.89\ \mu m$	$LF_{RED} = .63 \times 45 = 28.4\ \mu m\, cm$
GREEN:	$d_G = \frac{.55}{\sin 45} = 0.78\ \mu m$	$LF_G = .55 \times 45 = 24.8\ \mu m\, cm$
BLUE	$d_B = \frac{.47}{\sin 45} = 0.66\ \mu m$	$LF_B = .47 \times 45 = 21.2\ \mu m\, cm$

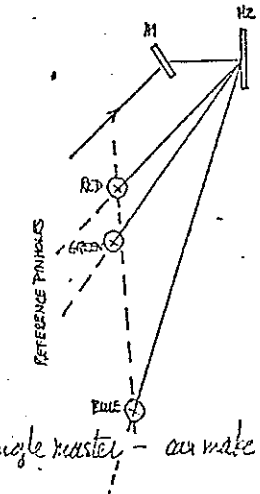
we have to make them all with one laser (He-Ne)

OPTION A: 1 object beam ($\theta O = 0°$, $s = 27.4$ cm) "spectrum" master

("Rum III" 1976 Walter + Benton Tamura, 1978) and 3 reference beams

	RED	GREEN	BLUE
$\sin \theta R =$	.63/.89	.63/.78	.63/.66
$\theta R =$	45°	54°	72°
$\frac{1}{S=27.4} - \frac{1}{RD} =$	.63/28.4	.63/24.8	.63/21.2
RD =	70.2 cm	91 cm	150 cm

FIG. 19)



advantages: single master – can make simple achromats

disadvantages: multiple exposures (to avoid gratings), have low diffraction efficiency ($\propto 1/\#$ exposures)

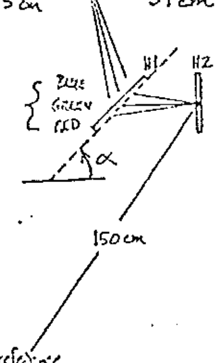
(13)

OPTION B: 3 object beams + 1 reference beam ($\theta R = 54°$; RD = 150)

("Skeletal Hand" 1977 Ocelin-Silver + Benton)

	RED	GREEN	BLUE
$\sin \theta O =$	$\frac{.63}{.89} - \sin 54$	$\frac{.63}{.78} - \sin 54$	$\frac{.63}{.66} - \sin 54$
$\theta O =$	−5.8°	0°	+ 8.2°
$\frac{1}{OD} - \frac{1}{150} =$	$\frac{.63}{28.4}$	$\frac{.63}{24.8}$	$\frac{.63}{21.2}$
OD =	34.5 cm	31 cm	27.4 cm

FIG. 20)
3 masters on 1 plate



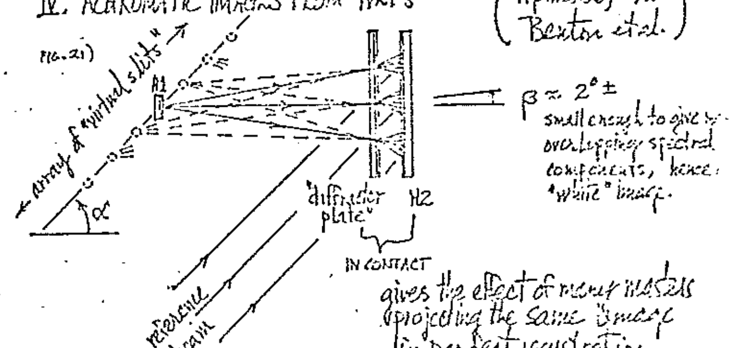
advantages: high diffraction efficiency (single exposure)

disadvantages: precise registration of image components is very difficult; achromats are impossible

IV. ACHROMATIC IMAGES FROM WLTS

("Aphrodite," 1977 Benton et al.)

FIG. 21)

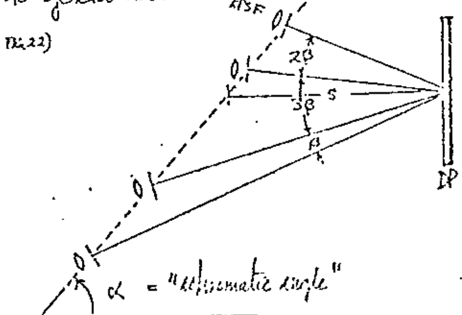


gives the effect of many masters projecting the same image in perfect registration

Key Element: the "diffractor plate" or multi-beam splitter; multi-focus off-axis zone plate

its generation:

FIG. 22)



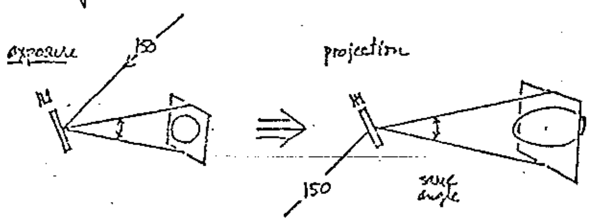
VIRTUAL FRAME

A useful (but usually imaginary) wire frame around the object, placed so that it will appear *in* the hologram plane, just at its edges. It is very useful for composition. Here, it must be placed 22 cm away from H1, and demagnified by a factor

$$MAG = \frac{31.4}{22} \approx 1.4$$

Thus, a 2⅞" x 3½" frame will "blow up" to 4" x 5" upon projection

BUT, the in-and-out magnification will be the square of the side-to-side magnification, or 2.0 in this case, so that spheres will come out looking like end-on footballs!

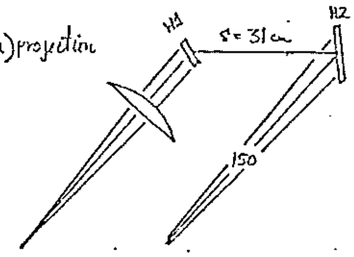


the images will also "roll" to follow the observer

WITH COLLIMATORS (on H1 only, RD = −PD = ∞)

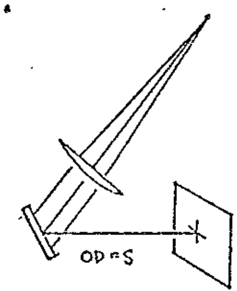
much easier, and *much* less distortion

FIG. 13a) projection



LF = 20

FIG. 13b) exposure

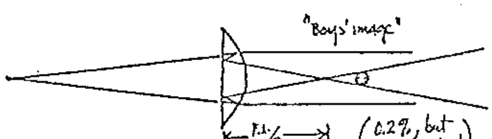


virtual frame:
same size as H2
little object/image distortion
(slight foreshortening)

ABOUT COLLIMATORS:

GLASS: plano-convex is nearly optimum spherical shape
use DIA = ¼ x focal length, or less
anti-reflection coatings

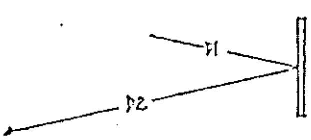
FIG. 14)



PLASTIC: cheaper in large sizes
have been used successfully
"laser quality" no "orange peel"?
no birefringence?
very fragile surfaces

HOLOGRAPHIC
will be aberrated, but useful (especially as a strip)

FIG. 15)

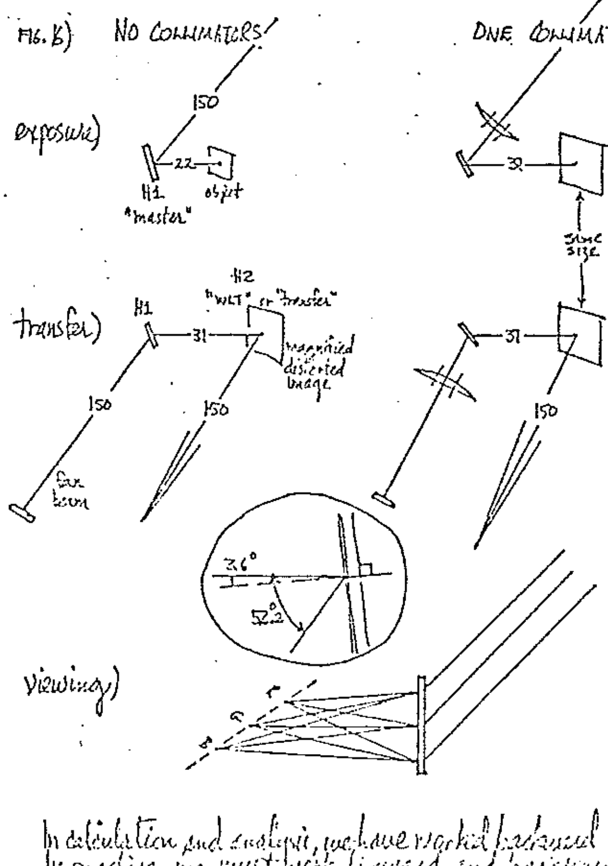


$$\frac{1}{D1} - \frac{1}{D2} = \frac{\lambda_2}{\lambda_1} \frac{1}{F.L.}$$

(the hardest holograms to make well!)

SO: TWO RAINBOW SYSTEMS (HeNe laser)

FIG. 16)



In calculation and analysis, we have worked backward. In practice, we must work forward and backward many, many times, and verify each step by experiment and observation.